

N.G.M COLLEGE (AUTONOMOUS): POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER - 2022
M.Sc. - MATHEMATICS
III SEMESTER
MAXIMUM MARKS: 70
TIME: 3 HOURS

PART III
21PMS313 - COMBINATORICS

SECTION – A

(10 X 1 =10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS.

(K1)

1. $P(n, r) =$ _____.
a) $\frac{np}{r!}$ b) $\frac{n!}{(n-r)!}$ c) $\frac{n!}{(n+r)!}$ d) $\frac{np!}{(n-r)!}$
2. The graph which has a pairing off of all the vertices is _____.
a) Complete b) Assignment c) edges d) bipartite
3. A simple tree is defined to be a tree with each vertex of degree _____.
a) $= 3$ b) $= 4$ c) ≤ 4 d) ≤ 3
4. No two octads can intersect in exactly _____ elements.
a) 1 b) 2 c) 3 d) 4
5. In Hadamard matrix, $AA' =$ _____.
a) $A'A$ b) IA' c) AI d) nl

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

6. Evaluate $P(7, 4)$.
7. State Assignment theorem.
8. Define simple network.
9. Write the inclusion – Exclusion principle for three sets.
10. Define Hadamard matrix.

SECTION –B**(5 X 4 = 20 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**11. a) Solve the recurrence relation $a_n = na_{n-1}$, subject to the condition $a_1 = 1$.

(OR)

b) How many solutions are there in non-negative integers of

(i) $x + y + z = 8$

(ii) $x + y + z + w = 18$?

12. a) 10 people meet and form 5 pairs. In how many ways can these 5 pairs be formed?

(OR)

b) State and prove the Harem Theorem.

13. a) If a_n denotes the n^{th} Fibonacci number then show that $a_{n+2} = a_n + a_{n-1} + \dots + a_1 + 2$

(OR)

b) Find a_4 and check the answer by writing down all the possible derangements of 1234.

14. a) Find the rook polynomial for an ordinary 4 X 4 board.

(OR)

b) Explain Leech's Lattice.

15. a) Show that there is no finite projective plane of order 6.

(OR)

b) Prove that the weight of each code word of G24 is a multiple of 4.

SECTION –C**(4X10 = 40 MARKS)****ANSWER ANY FOUR OUT OF SIX QUESTIONS****(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS****FROM Qn. No: 17 To 21) K4 & K5**16. Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$. (K4)17. Prove that $\exp(x) \cdot \exp(y) = \exp(x+y)$ using binomial theorem (K4)

18. State and prove exchange property (K4)

19. Obtain the recurrence relation (K5)

$$U_n = U_{n-1} + U_1 U_{n-2} + U_2 U_{n-3} + \dots + U_{n-2} U_1$$

20. If E, F are octads and $E \cap F = \emptyset$ then show that $(E + F)^1$ is also an octad. (K4)21. If A is square $(0,1)$ matrix and if A satisfies $AA' = (k - \lambda)I + \lambda J$ with $k > \lambda$ then prove that $AA' = (k - \lambda)I + \lambda J$ (K4)
