

**N.G.M COLLEGE (AUTONOMOUS): POLLACHI**  
**END-OF-SEMESTER EXAMINATIONS : NOVEMBER - 2022**  
**M.Sc. - MATHEMATICS** **MAXIMUM MARKS: 70**  
**III SEMESTER** **TIME: 3 HOURS**

**PART III**  
**21PMS313 - COMBINATORICS**  
**SECTION – A**

**(10 X 1 =10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS.**

**MULTIPLE CHOICE QUESTIONS.**

(K1)

1.  $P(n, r) = \underline{\hspace{2cm}}$ .  
a)  $\frac{np}{r!}$     b)  $\frac{n!}{(n-r)!}$     c)  $\frac{n!}{(n+r)!}$     d)  $\frac{np!}{(n-r)!}$
2. The graph which has a pairing off of all the vertices is                   .  
a) Complete    b) Assignment    c) edges    d) bipartite
3. A simple tree is defined to be a tree with each vertex of degree           .  
a)  $= 3$     b)  $= 4$     c)  $\leq 4$     d)  $\leq 3$
4. No two octads can intersect in exactly                    elements.  
a) 1    b) 2    c) 3    d) 4
5. In Hadamard matrix,  $AA' = \underline{\hspace{2cm}}$ .  
a)  $A'A$     b)  $|A'|$     c)  $|A|$     d)  $nI$

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.**

(K2)

6. Evaluate  $P(7, 4)$ .
7. State Assignment theorem.
8. Define simple network.
9. Write the inclusion – Exclusion principle for three sets.
10. Define Hadamard matrix.

**SECTION -B** **(5 X 4 = 20 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**11. a) Solve the recurrence relation  $a_n = na_{n-1}$ , subject to the condition  $a_1 = 1$ .

(OR)

b) How many solutions are there in non-negative integers of

(i)  $x + y + z = 8$ (ii)  $x + y + z + w = 18$  ?

12. a) 10 people meet and form 5 pairs. In how many ways can these 5 pairs be formed?

(OR)

b) State and prove the Harem Theorem.

13. a) If  $a_n$  denotes the  $n^{\text{th}}$  Fibonacci number then show that  $a_{n+2} = a_n + a_{n-1} + \dots + a_1 + 2$ 

(OR)

b) Find  $a_4$  and check the answer by writing down all the possible derangements of 1234.

14. a) Find the rook polynomial for an ordinary 4 X 4 board.

(OR)

b) Explain Leech's Lattice.

15. a) Show that there is no finite projective plane of order 6.

(OR)

b) Prove that the weight of each code word of G24 is a multiple of 4.

**SECTION -C****(4X10 = 40 MARKS)****ANSWER ANY FOUR OUT OF SIX QUESTIONS****(16<sup>th</sup> QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS****FROM Qn. No: 17 To 21) K4 & K5**16. Prove that  $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ . (K4)17. Prove that  $\exp(x) \cdot \exp(y) = \exp(x+y)$  using binomial theorem (K4)18. State and prove exchange property (K4)19. Obtain the recurrence relation (K5)

$$U_n = U_{n-1} + U_1 U_{n-2} + U_2 U_{n-3} + \dots + U_{n-2} U_1$$

20. If  $E, F$  are octads and  $E \cap F = \emptyset$  then show that  $(E + F)^1$  is also an octad. (K4)21. If  $A$  is square  $(0,1)$  matrix and if  $A$  satisfies  $AA' = (k - \lambda)I + \lambda J$  with  $k > \lambda$  then prove that  $AA' = (k - \lambda)I + \lambda J$  (K4)

\*\*\*\*\*