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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS311

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022

COURSE NAME: M.Sc.- MATHEMATICS | **MAXIMUM MARKS: 70**

SEMESTER: III | **TIME : 3 HOURS**

TOPOLOGY

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. Every finite point set in a Hausdorff space X is___________.
   (a) open (b) closed (c) compact (d) finite
2. A function $f : X \to Y$ is said to be continuous if for each open subset V of Y, the set $f^{-1}(V)$ is ______ of X.
   (a) open subset (b) closed subset (c) infinite (d) finite
3. A finite Cartesian product of connected spaces is__________.
   (a) compact (b) closed (c) connected (d) open
4. Every regular space with a countable basis is__________.
   (a) regular (b) compact (c) countable (d) normal
5. If X is completely regular, then X has a____________.
   (a) compactification (b) basis (c) Hausdorff space (d) none of these

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define standard topology on the real line.
7. Define norm of x.
8. State intermediate value theorem.
9. State Smirnov metrization theorem.
10. Define G_δ set in X.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Suppose that **C** is a collection of open sets of a topological space X such that for each open set U of X and each x in U, there is an element C of **C** such that $x \in C \subset U$. Prove that **C** is a basis for the topology of X.

(OR)

b) Let A be a subset of the topological space X and A' be the set of all limit points of A. Prove that $\bar{A} = A \cup A'$.

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12.a) State and prove the pasting lemma.

(OR)

b) State and prove the sequence lemma.

13.a) Prove that the union of a collection of connected subspace of X that have a point in common is connected.

(OR)

b) Prove that every compact subspace of a Hausdorff space is closed.

14.a) Prove that every compact Hausdorff space is normal.

(OR)

b) Prove that a subspace of a completely regular space is completely regular.

15.a) State and prove Tychonoff theorem.

(OR)

b) Let X be a completely regular space. Prove that there exists a compactification Y of X having the property that every bounded continuous map $f : X \to R$ extends uniquely to a continuous map of Y into R.

SECTION - C **(4 X 10 = 40 MARKS)**

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS)
K4 & K5

16. State and prove Urysohn metrization theorem. **K4**

17. Let Y be a subspace of X, A be a subset of Y and $\bar{A}$ denote the closure of A in X. Then prove that the closure of A in Y equals $\bar{A} \cap Y$. **K5**

18. Prove that the topologies on R^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on R^n. **K5**

19. Prove that the product of finitely many compact spaces is compact. **K4**

20. State and prove Tietze extension theorem. **K4**

21. State and prove Nagata-Smirnov metrization theorem. **K4**

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