(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS103

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: I **TIME : 3 HOURS**

COMPLEX ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

MULTIPLE CHOICE QUESTIONS.

1. A region is simply connected if its complement with respect to the extended plane is_______.
   (a) connected (b) disconnected (c) compact (d) closed
2. A real valued function is said to be harmonic if it satisfies__________.
   (a) C-R equations (b) Laplace's equation (c) difference equation (d) none of these
3. Convergence is uniform on every _____________ subset.
   (a) closed (b) open (c) compact (d) none of these
4. Each function in an equi-continuous family is itself_____________.
   (a) continuous (b) equi-continuous (c) compact (d) uniformly continuous
5. A function f(z) is said to be periodic with period T if__________.
   (a) f(z+T) = f(z) (b) f(z-T) = f(z) (c) f(zT) = f(z) (d) f(z/T) = f(z)

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. State Cauchy's integral formula.
7. State mean-value property.
8. Define entire function.
9. Mention one application of Poisson-Jensen formulas.
10. Define the order of the elliptic function.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that a region Ω is simply connected iff n(γ ,a) = 0 for all cycles γ in Ω and all points a which do not belong to Ω.

(OR)

b) State and prove Argument principle.

12. a) If u_1 and u_2 are harmonic in a region Ω, then prove that $\int u_1 * du_2 - u_2 * du_1 = 0$ for every cycle γ which is homologous to zero in Ω.

(OR)

b) State and prove Schwarz lemma.

(CONTD …. 2)

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13. a) State and prove Hurwitz theorem.

(OR)

b) State and prove Weierstrass theorem.

14.a) State the condition of total boundedness in terms of the original metric rather than in terms of the auxillary metric.

(OR)

b) Prove that a locally bounded family of analytic functions has locally bounded derivatives.

15.a) Prove that a discrete module consists either of zero alone, of the integral multiples nw of a single complex number $w \neq 0$, or of all linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with non-real ratio w_2 / w_1.

(OR)

b) Prove that an elliptic function without poles is a constant.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) State and prove the general Cauchy's theorem. **K4**

(OR)

b) State and prove Cauchy's residue theorem. **K4**

17. a) Derive Poisson's formula. **K5**

(OR)

b) State and prove the reflection principle. **K4**

18. a) State and prove Mitta-effler theorem. **K4**

(OR)

b) Derive Legendre's duplication formula. **K5**

19. a) Establish Jensen's formula. **K5**

(OR)

b) State and prove Arzela's theorem. **K4**

20. a) Prove that there exists a basis (w_1, w_2) such that the ratio $r = w_2 / w_1$ satisfies the following conditions: (i) $\operatorname{Im} r > 0$, (ii) $-1/2 < \operatorname{Re} r \leq 1/2$, (iii) $|r| \geq 1$, (iv) $\operatorname{Re} r \geq 0$ if $|r| = 1$. Also prove that the ratio r is uniquely determined by these conditions, and there is a choice of two, four, or six corresponding bases. **K5**

(OR)

b) Prove that the zeros $a_1, ..., a_n$ and poles $b_1, ..., b_n$ of an elliptic function satisfy $a_1 + ... + a_n \equiv b_1 + ... + b_n \ (Mod\ M)$. **K5**

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