(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022

COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: I **TIME : 3 HOURS**

ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)
MULTIPLE CHOICE QUESTIONS.

1. If $O(G) = p^2$ where p is a prime number, then G is___________.
   (a) abelian (b) sub group (c) cyclic (d) group
2. Commutative division ring is known as___________.
   (a) ring (b) abelian (c) group (d) field
3. If $a \in K$ is algebraic of degree n over F, then [F(a):F] =________.
   (a) F (b) F(a) (c) n (d) nF
4. If $\{v_i\}$ is an orthonormal set, then the vectors in $\{v_i\}$ are ____________.
   (a) linearly dependent (b) linearly independent
   (c) orthonormal (d) orthogonal
5. Any finite extension of a field of characteristic 0 is a_____________.
   (a) simple extension (b) finite extension (c) extension (d) none of these

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. State second Sylow's theorem.
7. State Gauss lemma.
8. Define primitive root of polynomial p.
9. State remainder theorem.
10. Define fixed field of a group G.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that N(a) is a subgroup of G.

(OR)

b) State and prove Cauchy's theorem.

12. a) State and prove the division algorithm.

(OR)

b) State and prove the Eisenstein criterion.

(CONTD …. 2)

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13. a) Prove that $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.

(OR)

b) Prove that for every prime number p and every positive integer m there exists a field having p^m elements.

14. a) If V is a finite-dimensional inner product space and if W is a subspace of V, prove that V is the direct sum of W and $W^{\perp}$.

(OR)

b) If $p(x) \in F[x]$ is irreducible and if a, b are two roots of p(x), then prove that F(a) is isomorphic to F(b) by an isomorphism which takes a onto b and which leaves every element of F fixed.

15. a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f^{'}(x)$ have a nontrivial common factor.

(OR)

b) If K is a finite extension of F, prove that G(K,F) is a finite group and its order, o(G(K, F)) satisfies $o\left(G\left(K,\ F\right)\right) \leq [K:F]$.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K4&K5

16. a) If p is a prime number and $p^{\alpha} \mid o(G)$ then, prove that G has a subgroup of order p^{α}. **K4**

(OR)

b) Prove that the number of p-sylow subgroups in G, for a given prime, is of the form 1 + kp. **K4**

17. a) Prove that a finite integral domain is a field. **K4**

(OR)

b) Prove that if R is a unique factorization domain, then so is R[x]. **K4**

18. a) If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, prove that [L:F] = [L:K] [K:F]. **K5**

(OR)

b) Prove that if F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then we can find elements a and b such that $1+\alpha+\alpha a^2+\beta b^2=0$ **K5**

19.a) Let V be a finite-dimensional inner product space. Then prove that V has an orthonormal set as a basis. **K4**

(OR)

b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field. **K4**

20.a) If F is of characteristic 0 and if a,b are algebraic over F, then prove that there exists an element $c \in F(a,b)$ such that F(a,b) = F(c). **K5**

(OR)

b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F. **K4**

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