(NO. OF PAGES: 3)

(FOR THE CANDIDATES ADMITTED **25UDA1A1**

DURING THE ACADEMIC YEAR 2025-2028 ONLY) **REG.NO. :**

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOV-2025

COURSE NAME: B.Sc. (CS with Data Analytics) **MAXIMUM MARKS: 75**
SEMESTER: I **TIME: 3 HOURS**

PART - III

25UDA1A1– Mathematical Foundation for Data Science

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

1. The statement "If it rains, then the ground is wet" is an example of which type of connective?

(A)Conjunction (B)Disjunction (C) Implication (D) Negation

2. If a relation R partitions a set A into disjoint subsets, these subsets are called:

(A) Reflexive pairs (B) Equivalence classes (C) Symmetric closures (D) Relation components.

3. In mathematical induction, to prove a statement P(n) for all-natural numbers (n):

(A) Verify P (0) and show $P(n) \Rightarrow P(n+1)$ (B) Verify only P (1)

(C) Show (P(n+1) ⇒ P(n)) (D) Substitute all possible values of (n)

4. The Travelling Salesman Problem (TSP) seeks to:

(A) Find the shortest path between two specific vertices

(B) Find a circuit that visits every edge exactly once

(C) Find a circuit that visits every vertex exactly once with minimum total weight

(D) Determine the number of components in a graph

5. In a tree, a vertex of degree 1 is called a**:**

(A) Root vertex (B) Internal vertex (C) Pendant vertex (D) Centre vertex

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES

6. Express the statement "Good food is not cheap "in a symbolic form.

7. Define Relation.

8. Check whether the function $f(x) = x^2 - 11$from R to R is 1-1?

9. Define Adjacency matrix of G with an example.

10. Define Rooted Tree with an example.

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SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

11. a) Find the truth value of $(P \wedge Q) \wedge (Q \vee R)$

(OR)

b) Show that $\sim (P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow (\sim P \vee Q)$

12. a) If R is the relation on $A = \{1, 2, 3\}$ such that $(a, b) \in R$,if and only if $a + b = even$, find the relational matrix M_R. Find also the relational matrices $R^{-1}, \bar{R}$ *and* R^2.

(OR)

b) Draw the relation graph for the following relations.

(i) R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)} on the set X = {1, 2, 3, 4}.

(ii) R_1 = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)} on the set Y= {1, 2, 3}.

13. a) Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x+4}$ is one-to-one and onto and hence find the inverse.

(OR)

b) Prove by mathematical induction that

$$1^1 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$$

14. a) Draw graphs representing problems of (a) two houses and three utilities; (b) four houses and four utilities, say, water, gas, electricity, and telephone.

(OR)

b) The number of vertices of odd degree in an undirected graph is even.

15. a) A tree with n vertices has (n-1) edges.

(OR)

b) Every connected graph has at least one spanning tree.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) Obtain the PDNF and PCNF of $(p \rightarrow (q \wedge r)) \wedge (\sim p \rightarrow (\sim q \wedge \sim r))$

(OR)

b) Give direct and indirect proof of $a \rightarrow b, c \rightarrow b, d \rightarrow (a \wedge c), d \Rightarrow b$.

17. a) Prove that the relation "Congruence modulo m" over the set of positive integers is an equivalence relation.

(OR)

b) Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$.Define the following relation R on A: $(a, b), R(a', b')$ iff $(ab' = a'b)$

18. a) If $S = \{1, 2, 3, 4, 5\}$ and if $f, g, h: S \to S$ are given by $f = \{(1,2), (2,1), (3,4), (4,5), (5,3)\}$ $g = \{(1,3), (2,5), (3,1), (4,2), (5,4)\}$, $h = \{(1,2), (2,2), (3,4), (4,3), (5,1)\}$

(a) Verify whether $fog = gof$.

(b)Explain why f and g have inverse but h does not.

(c)Find f^{-1} and g^{-1}.

(d) Show that $(fog)^{-1} = g^{-1}of^{-1} \neq f^{-1}og^{-1}$.

(OR)

b) Use mathematical induction to show that

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$, for n≥ 2.

19. a) Prove that a connected graph G is a Euler graph if and only if all vertices of G are of even degree.

(OR)

b) The maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$.

20. a) The number n of vertices of a full binary tree is odd and the number of pendant vertices of the tree is equal to $\frac{(n+1)}{2}$.

(OR)

b) Show that in a spanning tree, a connected graph of n vertices and e edges has n − 1 tree branches and e − n + 1 chords.

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