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(FOR THE CANDIDATES ADMITTED 25PMS207

DURING THE ACADEMIC YEAR 2025 -2026 ONLY) REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY 2026

M.Sc. - MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER II **TIME : 3 HOURS**

PARTIAL DIFFERENTIAL EQUATIONS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1. If the solution of $f(x, y, z , p\ q) = 0$ is given by two parameter system $f(x, y, z, a, b) = 0$ then the system form the _______________ integral.

a)singular b) complete c) general d) both (a) and (b)

2. If a linear differential equation can be written as the product of the linear factors of the form $(D + Aa + b)$ then it is said to be ____________.

a) an exact differential equation b) a homogeneous differential equation

c) reducible d) irreducible

3. A powerful and fundamental method for finding solutions to second-order linear partial differential equations (PDEs) is____________

a) Method of characteristics b) Method of separation of variables

c) Finite difference method d) Variational method

4. If the function $\psi(x, y, z)$ is a solution of Laplace's equation, then the one-parameter system of surfaces $\psi(x, y, z) = c$ is called ____________.

a) Stream surfaces b) Characteristic surfaces

c) Equipotential surfaces d) Isothermal surfaces

5. The Helmholtz equation obtained is ___________

a) $(\nabla^2 - \lambda^2)\phi = 0$ b) $\nabla^2\phi = 0$ c) $\nabla^2\phi = \lambda$ d) $(\nabla^2 + \lambda^2)\phi = 0$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Eliminate the arbitrary constant form the equation $z = (x + a)(y + b)$.

7. Define complementary function.

8. Write the procedure to be followed in applying the theory of integral transforms to the solution of partial differential equations.

9. Define Exterior Dirichlet boundary value problem.

10. Write the one-dimensional diffusion equation.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and fins their solution.

(OR)

b) Find a complete integral of the equation $p^2x + q^2y = z$.

12. a) If $\beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$is an arbitrary function of the single variable ξ, then prove that if $\beta_r \neq 0, u_r = exp\left(-\frac{\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$ is a solution of the equation $F(D, D') = 0$.

(OR)

b) Find a particular integral of the equation $(D^2 - D^{'})z = 2y - x^2$.

13. a) By separation of variables, show that the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ has solution of the form $A\exp\{\pm inx \pm inct\}$ where A and n are constants.

(OR)

b) Derive the solution of the one-dimensional diffusion equation.

14. a) Derive the elementary solution of Laplace's equation.

(OR)

b) Prove that the solution of a certain Neumann problem can differ from one another by a constant only.

15. a) Derive the elementary solution of the one-dimensional wave equation.

(OR)

b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r,\theta)$.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K4 (Or) K5)

16. a) Solve the partial differential equation $px^2 + qy^2 = z$ by Jacobi's method.

(OR)

b) Verify that the equation (i) $z = \sqrt{2x+a} + \sqrt{2y+b}$ (ii) $z^2 + \mu = 2\,(1+\lambda^{-1})(x+\lambda y)$ are both complete integrals of the partial differential equation$z = \frac{1}{p} + \frac{1}{q}$. Show that the complete integral (ii) is the envelope of the one-parameter subsystem obtained by taking $b = -\frac{a}{\lambda} - \frac{\mu}{1+\lambda}$ in the solution (i).

17. a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form.

(OR)

b) Solve the equation $r + s - 2t = e^{x+y}$.

18. a) Derive the solution of the equation $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$for the region $r > 0, z > 0$, satisfying the conditions: (i) $V \to 0$ as $z \to \infty$and $r \to \infty$. (ii) $v = f(r)$on $z = 0, r > 0$.

(OR)

b) Solve the equation $r + 4s + t + r\,t - s^2 = 2$.

19. a) A uniform insulated sphere of dielectric constant k and radius a carries on its surface of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 kn}{(2n+1)(kn+n+1)^2}$ to the electrostatic energy.

(OR)

b) A uniform circular wire of radius a charged with electricity of line density e surrounds grounded concentric spherical conductor of radius c. Determine the electrical density at any point on the conductor.

20. a) A thin membrane of grate extent extents is released from rest in the position $z = f(x,y)$. Determine the displacement at any subsequent time..

(OR)

b) The faces $x = 0, x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f(x)(0 \le x \le a)$. Determine the temperature at a subsequent time t.

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