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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2025-2026 ONLY)

25PMS206

REG.NO.

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY 2026

MSc.- MATHEMATICS — **MAXIMUM MARKS: 75**

SEMESTER II — **TIME : 3 HOURS**

MATHEMATICAL STATISTICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1. If X and Y are independent, then joint probability density function $f(x, y)$ =______

(A) $f(x) + f(y)$ (B) $f(x)\, f(y)$

(C) $f(x - y)$ (D) 0

2. If X has a standard normal distribution, then X^2 follows which of the following distributions?

(A) t distribution with 1 degree of freedom

(B) Chi-square distribution with 1 degree of freedom

(C) F distribution with 1 and 1 degrees of freedom

(D) Normal distribution with mean 0 and variance 1

3. An estimator is unbiased if__________

(A) Variance = 0 (B) Its variance is minimum

(C) Its expected value equals the parameter (D) It is consistent

4. For samples from two normal populations with sample variances $S_1{}^2$ and $S_2{}^2$, the estimator for the ratio $\sigma_1{}^2/\sigma_2{}^2$ is___________

(A) $(S_1 + S_2)/2$ (B) $S_1{}^2 - S_2{}^2$

(C) $(S_1 - S_2)^2$ (D) $S_1{}^2/S_2{}^2$

5. Rejecting the null hypothesis when it is true is known as______

(A) Type I error (B) Type II error

(C) Sampling error (D) Standard error

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define probability distribution function.

7. Write the formula for finding the sample mean and sample variance, if $X_1, X_2, \ldots X_n$ constitute a random sample.

8. Define consistent estimator.

9. Explain pivotal method.

10. What is an alternative hypothesis?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find the distribution function of the total number of heads obtained in four tosses of a balanced coin.

(OR)

b) If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} x + y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find the corresponding joint distribution function.

12. a) If $X_1, X_2, \cdots, X_n$ constitute a random sample from an infinite population which has the mean μ and the variance σ^2 then prove that $E(\bar{X}) = \mu$ and $var(\bar{X}) = \frac{\sigma^2}{n}$.

(OR)

b) Test the claim that the average gasoline consumption of this engine is 12.0 gallons per hour, if 16 one-hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a standard deviation of 2.1 gallons.

13. a) Show that $\bar{X}$ is a minimum variance unbiased estimator of the mean μ of a normal population.

(OR)

b) Use the method of moments, to obtain a formula for estimating the parameter α, if a random sample of size n from a uniform population with $\beta = 1$.

14. a) Construct a 95% confidence interval for the population mean μ, if a random sample of size $n = 20$ from a normal population with the variance $\sigma^2 = 225$ has the mean $\bar{X} = 64.3$.

(OR)

b) Find a 99% confidence interval for the difference between the actual proportions of the male and female voters who favor the candidate, if 132 of 200 male voters and 90 of 159 female voters favor a certain candidate running for a governor of Illinois.

15. a) Suppose that the manufacturer of a new medication wants to test the null hypothesis $\theta = 0.90$ against the alternative hypothesis $\theta = 0.60$. His test statistics is X, the observed number of successes (recoveries) in 20 trials, and he will accept the null hypothesis if $x > 14$, otherwise he will reject it. Find α and β.

(OR)

b) An oil company claims that less than 20% of all car owners have not tried its gasoline. Test this claim at the 0.01 level of significance if the random check reveals that 22 of 200 car owners have not tried the oil company's gasoline.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) If the probability density function of the random variable X is given by

$f(x) = \begin{cases} k \cdot e^{-3x} & for\ x > 0 \\ 0 & elsewhere \end{cases}$, then find k and $P(0.5 \leq x \leq 1)$

(OR)

b) If the joint probability density $(x, y) = \begin{cases} 4xy & for \quad 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases}$,

Find the marginal densities of X and Y and the conditional density of X given by $Y = y$.

17. a) Prove that $cov(X_r, X_s) = -\frac{\sigma^2}{N-1}$, if X_r and X_s are the rth and sth random variables of a random sample of size n drawn from the finite population $\{c_1, c_2, \cdots, c_N\}$.

(OR)

b) If U and V are independent random variables having chisquare distributions with v_1, and v_2 degrees of freedom, then prove that $F = \frac{U/v_1}{V/v_2}$ is a random variable having an F distribution, that is, a random variable whose probability density is given by

$$g(f) = \frac{\Gamma\left(\frac{v_1 + v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} \left(\frac{v_1}{v_2}\right)^{\frac{v_1}{2}} \cdot f^{\frac{v_1}{2}-1} \left(1 + \frac{v_1}{v_2} f\right)^{-\frac{1}{2}(v_1+v_2)}$$

for $f > 0$ and $g(f) = 0$ elsewhere

18. a) Show that $Y = \frac{1}{6}(X_1 + 2X_2 + 3X_3)$ is not a sufficient estimator of the Bernoulli parameter θ.

(OR)

b) Find joint maximum likelihood estimates of these two parameters, if $X_1, X_2, \cdots, X_n$ constitute a random sample of size n from a normal population with the mean μ and the variance σ^2.

19. a) Construct a 94% confidence interval for the difference between the mean lifetimes of two kinds of light bulbs, given that a random sample of 40 light bulbs of the first kind lasted on the average 418 hours of continuous use and 50 light bulbs of the second kind lasted on the average 402 hours of continuous use. The population standard deviations are known to be $\sigma_1 = 26$ and $\sigma_2 = 22$.

(OR)

b) In 16 test runs the gasoline consumption of an experimental engine had a standard deviation of 2.2 gallons. Construct a 99% confidence interval for σ^2, which measures the true variability of the gasoline consumption of the engine.

20. a) Suppose that 100 tires made by a certain manufacturer lasted on the average 21,819 miles with a standard deviation of 1,295 miles. Test the null hypothesis $\mu = 22{,}000$ miles against the alternative hypothesis $\mu < 22{,}000$ miles at the 0.05 level of significance.

(OR)

b) In comparing the variability of the tensile strength of two kinds of structural steel, an experiment yielded the following results: $n_1 = 13, s_1^2 = 19.2, n_2 = 16$, and $s_2^2 = 3.5$, where the units of measurement are 1,000 pounds per square inch. Assuming that the measurements constitute independent random samples from two normal populations, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative $\sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

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