(FOR THE CANDIDATES ADMITTED **24PMS417**

DURING THE ACADEMIC YEAR 2024-2025 ONLY) **REG.NO.**

N.G.M. COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY 2026

M.Sc.Mathematics **MAXIMUM MARKS: 75**

SEMESTER IV **TIME : 3 HOURS**

ALGEBRAIC TOPOLOGY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1. In a simply connected space X, any two paths having the same initial & final points are _______.

   a) homotopic b) path homotopic c) isomorphic d) homomorphic

2. A covering map $p: E \to B$ is ______.

   a) injective b) open & continuous c) continuous &surjective d) homeomorphism

3. If a space deformation retracts onto a subspace, then the two spaces are_______.

   a) Homeomorphic b) Isometric c) Homotopy equivalent d) Identical

4. Barycentric subdivision of a simplicial complex _______.

   a) changes homotopy type b) changes homology groups
   c) preserves homotopy type d) destroyssimplices

5. If X is a point, then $H_n(X) = 0$ for $n > 0$ and $H_0(X) \approx$_______.
   a) 0b) Z c) ℝd) $\bar{\bar{Z}}$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define homotopy.
7. What is meant by lifting of paths?
8. When do you say that A is a retract to X.?
9. Give an example for an abstract simplicial complex.
10. Write the sequence for chain complex.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If f is a path,and its path – homotopy equivalence relation is (f),then prove that the relations $\simeq$ and $\simeq_p$ are equivalence relations..

(OR)

b) Prove that the map $\hat{\alpha}$ is a group isomorphism.

12. a) If $p: E \to B$ and $p': E' \to B'$ are covering maps, then prove that $p\ X\ p': E\ X\ E' \to B\ x\ B'$is a covering map.

(OR)

b) Prove that the fundamental group of $S^1$is isomorphic to the additive group of integers.

13. a) State and Prove the Brouwer fixed-point theorem for the disc.

(OR)

b) Prove that the inclusion map $j: S^n \to \mathbb{R}^{n+1} - 0$ induces an isomorphism of fundamental groups.

14. a) Prove that the geometric realization of a finite simplicial complex is a polyhedron.

(OR)

b) If $g: K \to L$ is a simplicial map, then prove that the induced map $|g|: |K| \to |L|$ is continuous.

15. a) If $X = T$, the torus with the $\Delta -$ complex structured having one vertex, three edges a, b, c and two 2-simplices U and L, then find $H_n^{\Delta}(T)$.

(OR)

b) Prove that if a chain map between chain complexes induces homomorphisms between the homology groups of the two complexes.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(k4or k5)

16. a) State and prove the functorial properties.

(OR)

b) Prove the following:

i) Given $x \in X$, let e_x denote the constant path $e_x: I \to X$ carrying all of I to the point x. If f is a path in X from $x_0$to x_1, then $[f] * [e_{x_1}] = [f]$ and $[e_{x_0}] * [f] = [f]$.

ii) Given the path f in X from $x_0$to x_1, let $\bar{f}$ be the path defined by $\bar{f}(s) = f(1-s)$, then $[f] * [\bar{f}] = [e_{x_0}]$ and$[\bar{f}] * [f] = [e_{x_1}]$.

17. a) Prove that the map $p: \mathbb{R} \to S^1$ given by the equation $p(x) = (cos2\pi x, sin2\pi x)$ is a covering map.

(OR)

b) Let $p: E \to B$ be a covering map, let $p(e_0) = b_0$. Let the map $F: I \: X \: I \to B$ be continuous, with $F(0,0) = b_0$. There is a lifting of F to a continuous map $\bar{F} : I \: X \: I \to E$ such that $\bar{F}(0,0) = e_0$. If F is a path homotopy then prove that $\bar{F}$ is a path homotopy.

18. a) State and Prove the fundamental theorem of algebra.

(OR)

b) Let$f: X \to Y$ be continuous, let $f(x_0) = y_0$. If f is a homotopy equivalence, then prove that $f_*: \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is an isomorphism.

19. a) State and Prove the simplicial Approximation theorem.

(OR)

b) Prove that the Barycentric subdivision of a simplicial complex is itself a simplicial complex.

20. a) Prove that if X is nonempty and path connected then $H_0(X) \approx Z$ also prove that for any space$X$, $H_0(X)$ is a direct sum of $Z's$, one for each path-component of X.

(OR)

b) If two maps $f.g: X \to Y$ are homotopic, then prove that they induce the same homomorphism $f_* = g_*: H_n(X) \to H_n(Y)$.

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