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(FOR THE CANDIDATES ADMITTED **24PMS4E1**

DURING THE ACADEMIC YEAR 2024-2025 ONLY) **REG NO**

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2026

M.SC., MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: IV **TIME : 3 HOURS**

MATHEMATICAL METHODS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1. If the kernel becomes infinite at one or more points within the range of integration then the integral equation is called ______.
   a) symmetric b) singular c) Volterra d) Fredholm
2. The simplest singular integral equation is ________ equation.
   a) Fredholm b)Volterra c) Abel's d) Separable
3. The functional $v[y(x)] = \int_0^1 [(y')^2 + 12xy]dx, y(0) = 0, y(1) = 1$ can be extremized on _______.
   a) $y = x^2$ b) $x = y^2$ c) $x + y = 0$ d) $x - y = 0$
4. An extremal which satisfies the appropriate end conditions is often called as______.
   a) Euler function b) stationary function c)strictly maximum d) strictly minimum
5. Boundary value problems are approximated by another direct method which is not variational are called _______ method.
   a) Euler's b) Ritz c) Kantorovich's d) Galerkin's

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define Fredholm determinant.
7. Why do we say that an integral equation is singular?
8. When is the expression $Mdx + Ndy$ an exact differential?
9. Write the Hamilton-Jacobi equation.
10. What is the fundamental principle of Ritz method?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K3)

11. a) Find the resolvent kernel for the integral equation
$g(s) = f(s) + \lambda \int_{-1}^{1} (st + s^2t^2)\, g(t)dt.$

(OR)

b) Solve the integral equation $g(s) = 1 + \lambda \int_0^{\pi} [\sin(s+t)\, g(t)dt.$

12. a) Reduce the initial value problem y"(s)+ λy(s)=F(s) with the initial conditions $y(0) = 1, y'(0) = 0$ to a Volterra integral equation.

(OR)

b) Solve the integral equation $f(s) = \int_a^s \frac{g(t)}{(cost - \cos\ \ s)^{\frac{1}{2}}}\, dt\,,\ \ 0 \le a < s < b \le \pi.$

13. a) State and Prove the fundamental lemma of calculus of variation.

(OR)

b) Find the externals for the functional $v[y(x), z(x)] = \int_{x_0}^{x_1} F(y', z')\, dx.$

14. a)Test for an extremum for the functional

$$v = \int_0^a y'^3 dx,\ \ y(0) = 0, y(a) = b, a > 0, b > 0$$

(OR)

b) Prove that the total energy of a conservative system remains constant, when the system is in motion.

15. a) Investigate the functional $v[z((x,y)] = \iint [(\frac{\partial z}{\partial x} - y)^2 + (\frac{\partial z}{\partial y} + x)^2] dxdy$ for an extremum associated with the torsion of a cylinder or prism.

(OR)

b) Find a continuous solution of the equation $\Delta z = -1$ in the domain D, which is an isosceles triangle bounded by the straight lines $y = \pm\frac{\sqrt{3}}{3} x$ and $x = b$ which solution vanishes on the boundary of the domain.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(k4ork5)

16. a) Show that the integral equation $g(s) = f(s) + \left(\frac{1}{\pi}\right) \int_0^{2\pi} (\sin(s+t)) g(t)dt$ possesses no solution for $f(s) = s$ but it possesses infinitely many solutions when $f(s) = 1.$

(OR)

b) Solve the Fredholm integral equation g(s)= f(s) $+\lambda \int_0^1 (1 - 3st) g(t)dt$ and evaluate the resolvent kernal.

17.a) Solve the boundary value problem $y''(s) + A(s)y'(s) + B(s)y(s) = F(s)\ with$

$y(a) = y_0,\ y(b) = y_1$ and reduce to a Fredholm type integral equation

$y(s) = f(s) + \int k(s,t)y(t)dt.$

(OR)

b) Solve the following integral equation

i) $f(s) = \int_a^s \frac{g(t)dt}{[s^2-t^2]^\alpha},\quad 0 < \alpha < 1,\ \ a < s < b$

ii) $f(s) = \int_s^b \frac{g(t)dt}{[t^2-s^2]^\alpha},\quad 0 < \alpha < 1,\ \ a < s < b.$

18. a) Find the curve joining two points along which a particle falling from rest under the influence of gravity travels from higher to the lower point in the least time.

(OR)

b) Derive the differential equation of free vibrations of a string.

19. a)Derive the Weierstrass function

$E(x,y,p,y') = F(x,y,y') - F(x,y,p) - (y'-p)F_p(x,y,p).$

(OR)

b)Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = \{\varphi_1(x) + \varphi_2(y)\}(dx^2 + dy^2).$

20. a)Find a solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x,y)$ inside the rectangle D,

$0 \le x \le a,\ 0 \le y \le b$that vanishes on the boundary of D.

(OR)

b) Explain briefly Kantorovich's method.

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