(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2025 -2026 ONLY)

25UMS203

REG.NO.:

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2026

B.Sc. - MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: II **TIME: 3 HOURS**

PART-III

TRIGONOMETRY, VECTORCALCULUS AND FOURIER SERIES GEOMETRY

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS. **(K1)**

1) $z + \frac{1}{z} =$ ______.
   (A) $2\cos\theta$ (B) $2i\cos\theta$ (C) $2\sin\theta$ (D) $2i\sin\theta$

2) $\sin\theta =$ _____.
   (A) $\frac{e^{i\theta}-e^{-i\theta}}{2}$ (B) $\frac{e^{i\theta}-e^{-i\theta}}{2i}$ (C) $\frac{e^{i\theta}+e^{-i\theta}}{2i}$ (D) $\frac{e^{i\theta}+e^{-i\theta}}{2}$

3) $\int_0^a f(x)dx =$ ________, if *f(x)* is even function.
   (A) 0 (B) $\int_{-a}^{a} f(x)dx$ (C) $\frac{1}{2}\int_{-a}^{a} f(x)dx$ (D) $2\int_{-a}^{a} f(x)dx$

4) If $\phi(x,y,z) = x^2y + y^2x + z^2$, then $\nabla\phi$ is ______ at the point (1,1,1).
   (A) 8 (B) $\sqrt{22}$ (C) $3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ (D) $3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

5) By Green's theorem, $\oint_C \; Mdx + Ndy =$ _________.
   (A) $\iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$ (B) $\iint_S \left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dxdy$
   (C) $\iint_S \left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right) dxdy$ (D) $\iint_S \left(\frac{\partial N}{\partial y} + \frac{\partial M}{\partial x}\right) dxdy$

ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES.

6) Write the formula for $\cos 3\theta$ and $\sin 3\theta$.
7) Show that $\cosh^2 z - \sinh^2 z = 1$.
8) Write the Dirichlet conditions.
9) If $\vec{A}$ and $\vec{B}$ are irrotational, then show that $\vec{A} \times \vec{B}$ is solenoidal.
10) State Stoke's theorem.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11) (a) Expand $\frac{\sin 3\theta}{\sin\theta}$ in terms of $\cos\theta$.

(OR)

(b) If $\frac{\sin\theta}{\theta} = \frac{2165}{2166}$, show that θ is equal to $3^\circ 1'$ nearly.

12) (a) State and prove Euler's relation.

(OR)

(b) If $tanh\ \frac{u}{2} = \tan\frac{\alpha}{2}$, show that $u = \log\tan\left(\frac{1}{4}\pi + \frac{1}{2}\alpha\right)$.

13) (a)If $f(x) = \begin{cases} -x & \text{in} \quad -\pi < x < 0 \\ x & \text{in} \quad 0 \le x < \pi \end{cases}$

Expand *f(x)*as Fourier series in the interval $-\pi$ to π.

(OR)

(b) Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.

14) (a) Find the directional derivative of $xyz - x\,y^2z^3$ at the point (1, 2, -1) in the direction of the vector $\hat{\imath} - \hat{\jmath} - 3\hat{k}$.

(OR)

(b) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$then show that $\nabla \times (r^n\vec{r}) = 0$.

15) (a)Find$\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (x^2 - y^2)\vec{\imath} + 2xy\,\vec{\jmath}$ and C is the square bounded by the coordinate axes and the lines $x = a$ and $y = a$.

(OR)

(b) Find the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ using Green's theorem.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16) (a) Show that $-2^6cos^4\theta sin^3\theta = \sin 7\theta + \sin 5\theta - 3\sin 3\theta - 3\sin\theta$.

(OR)

(b) Find the expansion of $\cos n\theta$.

17) (a) Show that, if any one of the following relations is true, then the other two relations also true.

(i) $\sinh u = \tan\alpha$ (ii) $\cosh u = \sec\alpha$ (iii) $\tanh u = \sin\alpha$.

(OR)

(b)Show that $\cosh^4\theta = \frac{1}{2^3}[\cosh 4\theta + 4\cosh 2\theta + 3]$.

18) (a)If $f(x) = \begin{cases} x & \text{in the range} \quad (0,\pi) \\ 2\pi - x & \text{in the range} \quad (\pi, 2\pi) \end{cases}$

Express *f(x)* as a Fourier series in the range $(0, 2\pi)$.

(OR)

(b) Find a cosine series in the range 0 to π for $f(x) = \begin{cases} x & in \quad 0 < x < \frac{\pi}{2} \\ \pi - x & in \quad \frac{\pi}{2} < x < \pi \end{cases}$

19) (a) Find the equations of the tangent plane and normal to the surface $xyz = 4$ at the point (1,2,2).

(OR)

(b) Show that$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is irrotational and solenoidal.

20) (a)Evaluate $\iint (y^2z\,\vec{\imath} + z^2x\,\vec{\jmath} + x^2y\,\vec{k})\cdot d\vec{S}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ lying in the positive octant.

(OR)

(b) Verify Gauss's theorem for $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ taken over the region bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.

****