**(FOR THE CANDIDATES ADMITTED**

**DURING THE ACADEMIC YEAR 2025-2026 ONLY)** **SUB CODE: 25UMS2A2**

**REG.NO.:**

**N.G.M. COLLEGE (AUTONOMOUS): POLLACHI**
**END-OF-SEMESTER EXAMINATIONS: MAY-2026**
**B.Sc.- MATHEMATICS** **MAXIMUM MARKS: 75**
**SEMESTER: II** **TIME: 3 HOURS**

## PART-III
## MATHEMATICAL STATISTICS-II

**SECTION – A (10 X 1 = 10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS. (K1)**

1) If each of the two variables $X$ and $Y$ takes two values, 0, 1 with positive probabilities, then $r(X,Y)$ _____ $\Longrightarrow$ $X$ and $Y$ are independent.
   (A) $= 0$ (B) $\neq 0$ (C) $< 0$ (D) $> 0$
2) The standard error of sample variance is _____.
   (A) $\sqrt{\sigma^2/2n}$ (B)$\sqrt{\sigma^2/n}$ (C)$\sqrt{PQ/2n}$ (D)$\sigma^2\sqrt{2/n}$
3) In a Chi-square test of goodness of fit, the null hypothesis $H_0$ is accepted, if _____.
   (A) $\chi^2 > \chi^2_\alpha(n-1)$ (B) $\chi^2 \geq \chi^2_\alpha(n-1)$
   (C) $\chi^2 \leq \chi^2_\alpha(n-1)$ (D) $\chi^2 < \chi^2_\alpha(n-1)$
4) The statistic of the t-Test for testing the significance of an observed sample correlation is ___.
   (A) $t = \frac{r}{\sqrt{(1-r^2)}}\sqrt{(n-2)}$ (B) $t = \frac{r}{\sqrt{(1+r^2)}}\sqrt{(n-2)}$
   (C) $t = \frac{1}{\sqrt{(1-r^2)}}\sqrt{(n-2)}$ (D)$t = \frac{1}{\sqrt{(1+r^2)}}\sqrt{(n-2)}$
5) In the regularity conditions, the third order derivative $\frac{\partial^3}{\partial\theta^3}\log L$ exists such that $\left|\frac{\partial^3}{\partial\theta^3}\log L\right|$ is _____ $M(x)$ , where $E[M(x)] < K$, a positive quantity.
   (A) equal to (B) not equal to (C) less than (D) greater than

**ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES.**

6) Define Regression Analysis.
7) Write the steps involved in the normal test.
8) Find the unbiased estimate for Population mean($\mu$).
9) Write the assumptions for Student's t-Test.
10) Define an unbiased estimator.

**SECTION – B (5 X 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

11) (a) Calculate the correlation coefficient for the following heights (in inches) of fathers ($X$) and their sons ($Y$):

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | : | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | : | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

**(OR)**

(b) The ranks of same 16 students in Mathematics and Physics are as follows. Two numbers within brackets denote the ranks of the students in Mathematics and Physics:

(1, 1),(2, 10), (3, 3), (4, 4), (5, 5), (6, 7), (7, 2), (8, 6), (9, 8), (10, 11), (11, 15), (12, 9), (13, 14), (14, 12), (15, 16), (16, 13).

Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Physics.

12) (a) Write the procedure for testing of hypothesis.

**(OR)**

(b) In two large populations, there are 30 and 25 percent respectively of blue-eyed people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?

13) (a) The guaranteed average life of a certain type of electric light bulbs is 1000 hours with a standard deviation of 125 hours. It is decided to sample the output so as to ensure that 90 per cent of the bulbs do not fall short of the guaranteed average by more than 2.5 per cent. What must be the minimum size of the sample?

**(OR)**

(b) Illustrate the applications of Chi-Square distribution.

14) (a) The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

**(OR)**

(b) In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5 per cent level, given that the 5 per cent point of *F* for $n_1 = 7$ and $n_2 = 9$ degrees of freedom is 3.29.

15) (a) $X_1$, $X_2$, and $X_3$ is a random sample of size 3 from a population with mean value $\mu$ and variance $\sigma^2$. $T_1$, $T_2$, $T_3$ are the estimators used to estimate mean value $\mu$, where $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_2 - 4X_3$, and $T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)/3$.

(i) Are $T_1$ and $T_2$ unbiased estimators?
(ii) Find the values of $\lambda$ such that $T_3$ is unbiased estimator for $\mu$.
(iii) With this value of $\lambda$ is $T_3$ a consistent estimator?
(iv) Which is the best estimator?

**(OR)**

(b) Estimate $\alpha$ and $\beta$ in the case of Pearson's Type III distribution by the method of moments:
$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $0 \le x < \infty$.

**SECTION – C (5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)**

16)(a) The joint probability distribution of *X* and *Y* is given below:

| Y \ X | -1 | +1 |
|---|---|---|
| 0 | 1/8 | 3/8 |
| 1 | 2/8 | 2/8 |

Find the correlation coefficient between *X* and *Y*.

**(OR)**

(b) Obtain the equation of two lines of regression for the following data. Also obtain the estimate of *X* for *Y =70.*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | : | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | : | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

17) (a) A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.

**(OR)**

(b)Random samples of 400 and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5% level.

18)a)An insurance agent has claimed that the average age of policy – holders who insure through him is less than the average for all agents, which is 30.5 years. A random sample of 100 policy – holders who had insured through him gave the following age distribution:

| Age last birthday | : | 16-20 | 21-25 | 26-30 | 31-35 | 36-40 |
|---|---|---|---|---|---|---|
| No. of Persons | : | 12 | 22 | 20 | 30 | 16 |

Calculate the arithmetic mean and standard deviation of this distribution and use these values to test his claim at the 5% level of significant. (Z(1.645)=0.95)

**(OR)**

(b) The following figures show the distribution of digits in numbers chosen at random from a telephone directory:

| Digits | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | : | 1026 | 1107 | 997 | 966 | 1075 | 933 | 1107 | 972 | 964 | 853 | 10000 |

Test whether the digits may be taken to occur equally frequently in the directory.

19) a) Obtain the t-Test for Difference of Means.

**(OR)**

(b) (i) A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. Is this significant of correlation in the population?

(ii) Find the least value of r in a sample of 18 pairs of observations from a bi-variate normal population, significant at 5% level of significance.

20) (a)Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$,

(i) $E_\theta(T_n) \to \gamma(\theta), n \to \infty$ and (ii) $Var_\theta(T_n) \to 0, \text{as } n \to \infty$. Then prove that $T_n$ is a consistent estimator of $\gamma(\theta)$.

**(OR)**

(b) State and prove Cramer-Rao inequality.

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