QUESTION

(For 2023 onwards)

(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED **SUB CODE: 23UMS615**

DURING THE ACADEMIC YEAR 2023 - 2026 ONLY) **REG.NO. :**

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: MONTH AND YEAR APRIL 2026

COURSE NAME: BSC MATEHMATICS **MAXIMUM MARKS: 75**

SEMESTER: VI **TIME : 3 HOURS**

PART - III

SUBJECT CODE: 23UMS615 – TITLE: REAL ANALYSIS-II

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
(Objective Questions with four Multiple Choices) **(K1)**
(Qn. No. 1 - 5)

1 Let $f: S \to S$ be a function from a metric space (s,d) into itself. A point p in S is called a _____of f if f(p)=p.

(a) Fixed point (b) closed (c) open (d) accumulation point

2. Lt f be a real-valued function defined on a subset S of a metric Space M, and assume $a \in S$. Then f is said to have a ________ at a if there is a ball B(a) such that $f(x) \le f(a) \;\; for \; all \; x \; in \; B(a) \cap S$. (a)

(a) local minimum (b) local maximum (c) local Extrema (d) chain

3. “Assume f has a derivative at each point of an open interval (a,b) and assume that f is continuous at both end points a and b if f(a)=f(b) there is at least one interior point c at which f’(c)=0.” Refers to ______

(a) Taylor’s (b) Rolle’s (c) generalized mean value (d) Heine’s

4. Let $P = \{x_0, x_1, ...x_n\}$ be a partition of [a,b] and let t_k be a point in the subinterval $[x_{k-1}, x_k]$, a sum of the firm $S(P, f, \alpha) = \sum_{i=1}^{n} f(t_k)\Delta\alpha_k$ is called a _________sum of f with respect to α.

(a) Additive property (b) bounded variation (c) Total variation (d) Riemann-stieltjes

5. f Satisfies Riemann’s condition with respect to α on [a,b] if for every $\in > 0$, there exists a partition $P_\in$ such that P finer than $P_\in$ implies _____

(a) $0 \le U(P, f, \alpha) \le L(P, f, \alpha) < \in$ (b) $0 \ge U(P, f, \alpha) \ge L(P, f, \alpha) < \in$

(c) $0 \le U(P, f, \alpha) - L(P, f, \alpha) < \in$ (d) $0 \ge U(P, f, \alpha) L(P, f, \alpha) > \in$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**
(Qn. No. 6 - 10)

6. Define open and closed set.

7. If f is differentiable at c then prove that f is continuous at c.

(CONTD …. 2)

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8. Write the statement of mean value theorem

9. If f is monotonic on [a,b] then show that f is of bounded variation on [a,b].

10. Define step function

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(Qn. No. 11 to 15)Questions for Short Answers with internal choices – 2 questions from each unit. (K3)

11. a) Let $f:S\to T$ be a function . If $X\subseteq S$ and $Y\subseteq T$ then prove (i) $X=f^{-1}(Y)\Rightarrow f(X)\subseteq Y$ (ii) $Y=f(X)\Rightarrow X\subseteq f^{-1}(Y)$

(OR)

11 b) Prove that f is continuous iff inverse image of a open set is open.

12. a) Let f be defined on an open interval (a, b) and assume that f has a local minimum or a local maximum at an interior of (a,b) . If f has a derivative (finite or infinite) at c then prove that F '(c)must be zero

(OR)

12 b) If f is define on (a,b) and differentiable at a point (a,b) then there is a function f*(depending on f and on c) which is continuous at c and which satisfies the equation $f(x)-f(c)=(x-c)f^*(x)\quad\forall\quad x\in(a,b)$ with $f^*(c)=f'(c)$ conversely if there is a function f* , continuous at c which satisfies the above then prove that f is differentiable at c and $f'(c)=f^*(c)$

13. a) State and prove intermediate value theorem for derivatives

(OR)

13 b) If f is monotonic on [a,b] then prove that the set of discontinuous of f is countabl

14. a) If f and g are bounded variation on [a,b] then prove that $V_{f\pm g}\le V_f+V_g$ and $V_{f.g}\le AV_f+BV_g$

(OR)

14 b) If $f\in R(\alpha)$ and if $g\in R(\alpha)$ gεR(α) on [a,b] then prove that c_1f+c_2g ε R(α) on [a,b] (c1 & c2 are constants) and $\int_a^b(c_{1f+}c_2g)d\alpha=c_1\int_a^b f\ d\alpha+c_2\int_a^b g\ d\alpha$

15. a) State and prove Euler summation formula.

(OR)

15 b) State and prove formula for integration by parts. ;

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(Qn. No. 16 to 20 Questions for Long Answers with internal choices – 2 questions from each unit. (K4 (Or) K5)

16. a) State and prove Heine's theorem

(OR)

16 b) Prove that a metric space S is connected iff every two-valued function on S is constant.

17. a) State and prove chain rule for differentiation.

(OR)

(CONTD …. 3)

17 b) Assume that f and g are defined on (a,b) and differentiable at c .then prove that $f+g$, $f-g$ and $f.g$ are also differentiable at c. this is also true of $\frac{f}{g}$ if $g(c)\neq 0$ the derivatives at c are given by the following formulas:

(i) $(f\pm g)'(c)=f'(c)\pm g'(c)$

(ii) $(f.g)'(c)=f(c)g'(c)+f'(c)g(c)$

(iii) $(\frac{f}{g})'(c)=\frac{g(c)f'(c)-f(c)g'(c)}{g(c)^2}$ $provided\ g(c)\neq 0$

18. a) State and prove Taylor's theorem.

(OR)

18 b) State and prove generalized mean value theorem.

19. a) State and prove additive property of total variation.

(OR)

19 b)` Assume that $c\in(a,b)$ If two of the three integrals in the below equation exists, then prove that the third also exists and we have $\int_a^c f\ d\alpha+\int_c^b f\ d\alpha=\int_a^b f\,d\alpha$

20. a) Assume that $\alpha\uparrow$ on [a,b] then prove that the following three statements are equivalent

$(i)\ f\in R(\alpha)\ on\ \ [a,b]$

(ii) f Satisfies Riemann's condition with respect to α on [a,b]

(iii) $I(f,\alpha)=I(f,\alpha)$

(OR)

20 b) Assume $f\in R(\alpha)$ on [a,b] and assume that α has a continuous derivative α' on [a,b] then prove that the Riemann integral $\int_a^b f(x)\,d\alpha(x)=\int_a^b f(x)\alpha'(x)dx$.

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**** PL USE TABLE AND FORMULA FORMATS FOR ACCOUNTS AND STATISTICS QNS AND MATHEMATICS AND PHYSICS DEPARTMENTS.***