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(FOR THE CANDIDATES ADMITTED **SUB CODE: 23UMS614**

DURING THE ACADEMIC YEAR 2023 - 2024 ONLY) **REG.NO. :**

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY 2026

B.Sc. -Mathematics **MAXIMUM MARKS: 75**

SEMESTER : VI **TIME : 3 HOURS**

PART - III

LINEAR ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1 In which the following statement is true

(a) A product of invertible matrices is invertible

(b) An elementary matrix is invertible

(c) A square matrix with either a left or right inverse is invertible

(d) all.

2 ______ of a finite dimensional vector space as the number of elements in a basis for V

(a) Ordered basis (b) dimension (c) Standard basis (d) basis

3. If V is finite-dimensional, the _____of T is the dimension of the null space of T

(a) Rank (b) isomorphic (c) nullity (d) invertible

4. In vector space of dimension n, subspace of dimension ______is called a hyper space.

(a) n (b) $n-1$ (c) $n+1$ (d) 1

5. If V is a vector space, a_______ in V is a maximal proper subspace of V

(a) Dual space (b) Transpose (c) relative to the ordered basis (d) hyper space

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define elementary matrix.
7. Define subspace.
8. Write about Annihilator.
9. What is called a linear functional?
10. Define transpose of a matrix.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find the inverse of $A=\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(OR)

11 b) Let $A = A_1, A_2, ...A_K$ where $A_1, A_2, ...A_K$ are $n \times n$ matrices .Then prove that A is invertible if and only if each A_i is invertible.

12. a) Prove that a non empty subset W of V is a subspace of if and only if for each pair of vectors α, β in W and each scalar c in F, the vector $c\alpha + \beta$ is again in W

(OR)

b) Let R be a non-zero row-reduced echelon matrix. Then prove that the non-zero row vectors of R form a basis for the row space of R

ETHICAL PAPER **(CONTD..2)**

13. a) Let V be a finite-dimensional vector space over the field F and let $\{a_1, a_2, \ldots\ldots\ldots a_n\}$ be an ordered basis for V. Let W be a vector space over the same field V and let $\beta_1, \beta_2, \ldots\ldots\beta_n$ be any vector in W. Then prove that there is precisely one linear transformation T from V in to W such that $T_{\alpha_i} - \beta_j, \quad j = 1,2,\ldots\ldots\ldots n$

(OR)

b) Prove that let V and W be vector space over field F and let T be a linear transformation from V in to W. Suppose that V is finite-dimensional then $rank(T) = nullity(T) = \dim V$

14. a) Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space F. For each pair of ordered bases B , B' for V and W respectively , the function which assigns to a linear transformation T its matrix relative to B,B' is an isomorphism between the space L(V,W) and the space of all $m \times n$ matrices over the field F

(OR)

14 b) Let V be a finite-dimensional vector space over the field F, and let $B = \{\alpha_1, \alpha_2, \ldots\ldots\alpha_n\}$ be a basis for V. Then prove that there is a unique dual basis $B^* = \{f_1, f_2, \ldots\ldots f_n\}$ for V* such that $f_i(\alpha_i) = \delta_i$ for each linear functional f on V we have $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and for each vector α in V we have $\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i$

15. a) If f and g are linear functionals on a vector space V, then prove that g is a scalar multiple of f iff the null space of g contains the null space of f

(OR)

b) Let A be any $m \times n$ matrix over the field F. Then prove that the row rank of A is equal to the column rank of A

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K4 (Or) K5)

16. a) Solve the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 4 & 0 & -1 \\ 2 & 6 & -1 & 5 \end{bmatrix}$ by using elementary row operations

(OR)

16 b) For an $n \times n$ matrix A, prove that the following are equivalent :
(i) A is invertible
(ii) The homogeneous system $AX = 0$ has only the trivial solution $X = 0$
(iii) The system of equations $AX = Y$ has a solution X for each $n \times 1$ matrix Y.

17. a) Prove that any two bases of a finite dimensional vector space V have the same number of elements.

(OR)

17 b) If W_1 and W_2 are finite finite-dimensional subspace of a vector space V, then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$

18. a) Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V Then Prove that $\dim W + \dim W^* = \dim V$

(OR)

18 b) Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then prove that the vector space $L(V,W)$ is finite-dimensional and has dimension mn.

19. a) Let W be the subspace of R^5 which is spanned by the vectors $\alpha_1 = (2,-2,3,4,-1), \alpha_2 = (0,0,-1,-2,3), \alpha_3 = (-1,1,2,5,2), \alpha_4 = (1,-1,2,3,0)$
Describe W^0 and the annihilator of W

(OR)

19 b) Let V be a finite-dimensional vector space over the field F and let W be a subspace of V then prove that $\dim W + \dim W^0 = \dim V$

20.a)Prove that if f is a non-zero linear functional on the vector space V, then prove that the null space of f is a hyperspace in V. Conversely every hyperspace in V is the null space of a (not unique) non-zero linear functional on V

(OR)

b) If S is any subset of a finite dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S.

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