**(FOR THE CANDIDATES ADMITTED** **23UMS6E4**

**DURING THE ACADEMIC YEAR 2023 - 2026 ONLY)** **REG.NO. :**

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**

**END-OF-SEMESTER EXAMINATIONS: MAY P2026**

**B.SC. MATHEMATICS** **MAXIMUM MARKS: 75**

**SEMESTER: VI** **TIME: 3 HOURS**

## PART - III

## 23UMS6E4 – THEORY OF NUMBERS

### SECTION – A (10 X 1 = 10 MARKS)

**ANSWER THE FOLLOWING QUESTIONS.** **(K1)**

1. Find $2^{35}$-1 is divisible by -----?
   a) 35 b) 30 c) 31 d) 33
2. "Let p denote a prime. If p does not divide a then prove that $a^{p-1} \equiv 1 (\mathrm{mod}\ p)$.
   For every integer a, $a^p \equiv a\ (\mathrm{mod}\ p)$," refers _______________theorem
   (a) Fermat's (b) Fermat's Little (c) Wilson's (d) Euclid's
3. 37 ≡_____ (mod 12)
   (a) 0 (b) 1 (c) -1 (d) 2
4. Find $\sigma$ (12) =?
   (a) 24 (b) 28 (c) 32 (d) 48
5. Every even number larger than 2 is the sum of two _____
   (a) Odd (b) even (c) primes (d) Mersenne primes

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES** **(K2)**

6. Find gcd (2a+3, 4a+5).
7. Write the statement of Wilson's theorem.
8. Define inverse of a modulo c.
9. Find $\sigma$ (180).
10. Define primitive root modulo m.

### SECTION – B (5 X 5 = 25 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.** **(K3)**

11. a)Given integers a and b, not both of which are zero, prove that there exist integers x and y such that $\gcd(a,b) = ax + by$.

**(OR)**

b) State and prove Division algorithm.

12. a) ) Prove that the product of any n consecutive positive inters is divisible by the product of the first n positive integers.

**(OR)**

b) If $s$ integer $r_1, r_2, \ldots\ldots\ldots\ldots r_s$ form a reduced residue system modulo m then prove that $s = \phi(m)$.

13. a) State and prove Chinese remainder theorem.

**(OR)**

13 b) State and prove Euler's theorem

14. a) State and prove Mobius inversion theorem.

**(OR)**

b) Prove that $\phi(n), d(n), \sigma(n),$ and $\mu(n)$ are multiplicative arithmetic functions.

15. a) Prove that for each prime p, there exist primitive roots modulo p.

**(OR)**

b) If $f(x) = \dfrac{x}{\log x}$, then prove that $f(x)$ is increasing for $x > e$,

$f(x-2) > \dfrac{1}{2} f(x) \;\; for \;\; x \geq 4$, $f\left(\dfrac{x+2}{2}\right) < \dfrac{15}{16} f(x) \;\; for \;\; x \geq 8$.

**SECTION – C (5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K4 (Or) K5)**

16. a) State and prove Euclid division algorithm.

**(OR)**

b) State and prove fundamental theorem of arithmetic

17. a)Prove that the quadratic congruence $x^2 + 1 \equiv 0(\text{mod} p)$, where p is an odd prime, has a solution iff $p \equiv 1(\text{mod } 4)$

**(OR)**

b) Prove that $a_n = \binom{2n-2}{n-1} / n$

18. a) Prove that the congruence $(m-1)! \equiv -1(\text{mod } m)$ holds ⇔ m is a prime

**(OR)**

b) If f(x) is a polynomial of degree n with integral coefficients (that is if $f(x) = a_0 x^n + a_1 x^{n-1} + \ldots\ldots. + a_n$) and if p is a prime such that $p \neq a_0$, then prove that the congruence $f(x) \equiv 0(\text{mod } p)$ has at most n mutually incongruent solutions modulo p.

19. a) Prove that $\sum_{d/n} \phi(d) = n$

**(OR)**

b) If $n = p_1^{\alpha_1} \ldots\ldots\ldots p_1^{\alpha_s}$ then prove that $d(n) = (\alpha_1 + 1)(\alpha_2 + 1)\ldots\ldots\ldots(\alpha_s + 1)$ and $\sigma(n) = \dfrac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \dfrac{p_2^{\alpha_2+1}}{p_2 - 1} \ldots\ldots\ldots\ldots\ldots \dfrac{p_s^{\alpha_s+1} - 1}{p_s - 1}$.

20. a) Prove that for $x \geq 8$, $\dfrac{\log 2}{4} . \dfrac{x}{\log x} < \pi(x) < 30(\log 2)\dfrac{x}{\log x}$.

**(OR)**

b) Prove that $\lim_{x \to \infty} \dfrac{\pi(x)}{x} = 0$.

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