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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS314

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025
M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: III **TIME : 3 HOURS**

GRAPH THEORY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. An every edge is a cut edge then a connected graph is a _____.
   a) tree b) spanning tree c) tour d) cotree
2. A connected graph that has no cut vertices is a _____.
   a) walk b) tree c) block d) path
3. The number of edges in a minimum edge covering of G is _____.
   a) independence number b) covering number
   c) edge covering number d) edge independence number
4. Every $k-$chromatic graph has at least k vertices of degree _____.
   a) at least k b) at least $k-1$ c) at most $k+1$ d) at most $k-1$
5. _____ contains a vertex from which every other vertex is reachable by a directed path of length at most 2.
   a) Hamilton path b) Tournament c) Hamilton cycle d) Directed cycle

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. What is contraction of an edge of G?
7. What is meant by tour?
8. State marriage theorem.
9. Define edge chromatic number.
10. What is meant by Jordan curve?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

(OR)

b) Prove that an every connected graph contains a spanning tree.

12. a) If G is a block with $v \geq 3$, then prove that any two edges of G lie on a common cycle.

(OR)

b) If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, then prove that G is Hamiltonian.

13. a) Prove that an every 3-regular graph without cut edges has a perfect matching.

(OR)

b) If $\delta > 0$, then prove that $\alpha' + \beta' = v$.

(CONTD……..2)

14. a) Let G be a connected graph that is not an odd cycle, then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

(OR)

b) Prove that in a critical graph, no vertex cut is a clique.

15.a) If G is a connected plane graph, then prove that $v - \epsilon + \varphi = 2$.

(OR)

b) Prove that if a digraph D contains a directed path of length $\chi - 1$.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Prove that a graph is bipartite if and only if it contains no odd cycle.

(OR)

b) Prove that a vertex v of a tree G is a cut vertices of G if and only if $d(v) > 1$.

17. a) Prove that $k \leq k' \leq \delta$.

(OR)

b) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

18. a) State and prove the Hall's theorem.

(OR)

b) Prove that a matching M in G is a maximum matching if and only if G contains no M −augmenting path.

19. a) State and prove the Brook's theorem.

(OR)

b) For any positive integer k, then prove that there exists a k −chromatic graph containing no triangle.

20. a) Let G be a nonplanar connected graph that contains no subdivision of k_5 or $k_{3,3}$ and has as few edges as possible, then prove that G is simple and 3 −connected.

(OR)

b) Prove that if D is strict and $\min[\delta, \delta'] \geq v/2 > 1$, then D contains a directed Hamilton cycle.

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