(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS313

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025
M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: III **TIME : 3 HOURS**

COMBINATORICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. The value of $p(8,2) =$ _______.
   a) 840 b) 56 c) 57 d) 66
2. If 10 people meet and form 5 pairs then the number of ways can these 5 pairs is ______.
   a) 945 b) 560 c) 957 d) 966
3. The number of n digit sequence are formed using the integers 0,1,2,3 is _______.
   a) n b) 4^n c) 4^{n-1} d) $4^n/n$
4. A code will correct all sets of h or fewer errors if any two words differ in at least ____ places.
   a) $h+1$ b) $h-1$ c) $2h+1$ d) $2h-1$
5. The seven-point plane is an example of _______.
   a) $S(2,3,9)$ b) $S(2,9,7)$ c) $S(2,3,7)$ d) $S(2,3,n)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Write the recurrence formula to find $f(n,k)$.
7. Find the number of ways can the 20 towns be put in to 5 groups of 4 towns.
8. If there are 4 colours available and $k = 6$, how many colourings of the golf balls are possible?
9. Find the rook polynomial non-interfering $2\ X\ 2$ block.
10. Define Steiner system.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Solve the recurrence relation $a_n = a_{n-1}$, subject to the condition $a_1 = 1$.

(OR)

b) 30 girls, including Miss U.K., enter a Miss World competition. The first 6 places are announced. **(i)** How many different announcements are possible?
(ii) How many if Miss U.K. is assured of a place in the first six?

12. a) Find an optimum solution for the following assignment problem:

	A	B	C	D
a	6	8	2	7
b	5	8	13	9
c	2	7	8	9
d	4	11	7	10

(OR)

(CONTD.........2)

12 b) Let $\{a_1, \ldots\ldots a_n\}$ and $\{b_1, \ldots\ldots b_{n+1}\}$ be two assignable sets, not necessarily disjoint then prove that for some $i \leq n+1, \{a_1, \ldots\ldots a_n, b_i\}$ is also an assignable set.

13.a) Suppose that a_1 and a_2 are given and that $a_n = Aa_{n-1} + Ba_{n-2}$ $(n \geq 3)$ holds. Prove that if the roots α, β of the equation $x^2 = Ax + B$ are distinct then prove that $a_n = K_1\alpha^n + K_2\beta^n$ where the constants K_1, K_2 are determined uniquely by a_1 and a_2.

(OR)

b) Define s_n and evaluate the value of s_5.

14. a) Find the rook polynomial for an ordinary $4 \; X \; 4$ board.

(OR)

b) Prove that each normalized Hadamard matrix A of order $4m \geq 8$ yields a $(4m-1, 2m-1, m-1)$ configuration.

15. a) Prove that the number of m-element sets in an $S(l, m, n)$ is $\binom{n}{l} / \binom{m}{l}$

(OR)

b) Prove that no two octads can intersect in exactly 3 elements.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.

(OR)

b) An eight-man committee is to be formed from a group of 10 Welshmen and 15 Englishmen. In how many ways can the committee be chosen if
(i) the committee must contain 4 of each nationality
(ii) there must be more Welshmen than Englishmen
(iii) there must be at least two Welshmen?

17.a) Prove that if a graph has $2n$ vertices, each of degree $\geq n$, then the graph has a perfect matching.

(OR)

b) State and prove the Konig-Egervary max-min theorem.

18. a) Suppose that n jobs have been assigned to n people. In how many ways can they be reassigned the following day so that no person is given the same job as before?

(OR)

b) Prove that $f(n, k) = \binom{n+k-1}{k}$.

19. a) The manager of a firm has 5 employees to be assigned to 5 different jobs. The men are A, B, C, D, E and the jobs are a, b, c, d, e. He considers that A is unsuited for jobs b and c, B unsuited for a and c, C unsuited for b, d and e, D suited for all and E unsuited for d. In how many ways can he assign the jobs to men suited to them?

(OR)

b) State and prove the Fisher theorem.

20.a) In $S(5,8,23)$; prove the following
(i) the number of octads is 759
(ii) each element of B lies in 253 octads
(iii) each pair of elements lies in 77 octads
(iv) each triple of elements lies in 21 octads
(v) each tetrad of elements (i.e. each four-element subset of B) lies in 5 octads
(vi) each quintuple of elements lies in just 1 octad.

(OR)

b) Prove that no dodecad contains an octad.

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