(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS312

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2025

M.Sc MATHEMATICS | **MAXIMUM MARKS: 75**

SEMESTER: III | **TIME: 3 HOURS**

FUNCTIONAL ANALYSIS

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. $\psi(x+u, y) =$ ______________
   a) $\psi(x,y)+\psi(u,y)$ b) $\psi(xy)+\psi(uy)$ c) $\psi(x+y,u)$ d) $\psi(x+y,u+y)$
2. Interior of a proper subspace of a normed linear space is ________.
   a) 2 b) 1 c) can't find d) empty
3. For every $x \in X$, $\|x\|^2 =$ _________________.
   a) $|\langle x,u\rangle|$ b) $\sum_{u\in E}|\langle x,u\rangle|$ c) $\sum_{u\in E}|\langle x,u\rangle|^2$ d) $\sum_{u\in E}|\langle x,u\rangle|^n$
4. A function $\wp: X \to \mathbb{R}$ is said to be a Convex Functional if _____________ .
   a) $\wp(x,y) \le \wp(x)+\wp(y)$ b) $\wp(x,y) = \wp(x)+\wp(y)$
   c) $\wp(x,y) \ne \wp(x)+\wp(y)$ d) $\wp(x,y) \ge \wp(x)+\wp(y)$
5. Let X and Y be normed linear spaces, and $A: X \to Y$ be a linear operator. If A is an open map, then A is ______________
   a) Injective b) Surjective c) equivalence d) closed

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define: Normed Linear Space.
7. State the Riesz Lemma.
8. What is meant by Transpose of an Operator?
9. Prove that $f(ix) = if(x)$
10. State the Bounded Inverse Theorem.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11. a) State and Prove the Holder's Inequality in $\mathcal{F}(\mathbb{N},\mathrm{K})$.

(OR)

b) Let X be a finite dimensional normed linear space of dimension $\kappa < \infty$, and let $E = \{u_1, u_2, \dots u_k\}$ be a basis of X. Consider the norm $\|x\|_E := \max\{|f_1(x)|, \dots, |f_k(x)|\}, x \in X$, on X, where $f_1, f_2, \dots, f_k$ are the coordinate functionals on X associated with E. Then prove that $\|.\|_E$ is a Banach Space.

12. a) State and prove Baire Category Theorem.

(OR)

b) State and prove Projection Theorem.

13. a) Let X and Y be normed linear space. If Y is a Banach Space, then prove that $\mathcal{B}(X,Y)$ is a Banach Space. In particular, prove that the dual of the normed linear space is a Banach Space.

(OR)

b) State and prove Riesz – Fischer Theorem.

(CONTD……….2)

14. a) State and prove Hahn – Banach Extension Theorem.

(OR)

b) Let X and Y be a normed linear space, and $A \in \mathcal{B}(X,Y)$. Then Prove that $\|A'\| = \|A\|$.

15. a) Let Z be a closed subspace of a normed linear space X. Then prove that the quotient map $\eta: X \to X/Z$ is a linear, surjective, continuous and open map.

(OR)

b) State and prove Open Mapping Theorem.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) State and prove Polarization Identities.

(OR)

b) Prove that any two norms on a finite dimensional linear space are equivalent.

17. a) Prove that a Banach Space cannot have a denumerable basis.

(OR)

b) State and Prove Henie – Borel Theorem.

18. a) Let X and Y be normed linear spaces and $A : X \to Y$ be a linear operator. Then prove that the following are equivalent.

i) A is continuous at 0.

ii) A is continuous at $x \in X$.

iii) A is uniformly continuous.

iv) There exists $c > 0$such that $\|Ax\| \leq c\|x\|$ for all $x \in X$.

v) $\{Ax : x \in X, \|x\| = 1\}$ is bounded subset of Y.

vi) For every bounded set $E \subseteq X$, the set $\{Ax : x \in E\}$ is bounded in E.

(OR)

b) i) State and prove Bessel's Inequality.

ii) Let E be an orthonormal set in an inner product space X. Then prove that for every $x \in X$, the set $E_x := \{u \in E : \langle x, u\rangle \neq 0\}$ is a countable set.

19. a) Let X be a linear space over $\mathbb{C}$. Then prove the following:

i) Let f be a complex – linear functional on X and $\varphi : X \to \mathbb{R}$ be defined by $\phi(x) = Re\ f(x), x \in X$.

Then ϕ is a real – linear functional on X, and $f(x) = \phi(x) - i\phi(ix), x \in X$.

ii) Let ψ be a real – linear functional on X and $f : X \to \mathbb{C}$ be defined by $f(x) = \psi(x) - i\psi(ix), x \in X$. Then f is a complex – linear functional on X.

iii) Let $v : X \to \mathbb{R}$ be a seminorm on X and f be a complex – linear on X. Then

$$|f(x)| \leq v(x), \forall\ x \in X \Leftrightarrow |Re\ f(x)| \leq v(x), \forall\ x \in X.$$

(OR)

b) Let X be a normed linear space and Ω be a dense subset of X. Then prove that X is linearly isometric with a subspace of $l^{\infty}(\Omega)$.

20. a) State and prove Uniform Bounded Principle.

(OR)

b) State and prove Closed Graph Theorem.

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