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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS311

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2025

M.Sc.-MATHEMATICS — MAXIMUM MARKS: 75

SEMESTER:III — TIME: 3 HOURS

TOPOLOGY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. The set X for which a Topology $\mathcal{T}$ has been specified is called a __________.
   a) Topological Range b) Topological Space c) Topological Value d) Topology
2. If X is a topological space, X is said to be _______ if there exists a metric d on the set X that induces the topology of X.
   a) Metrizable b) Metric Space c) Uniform Metric d) Bounded Metric
3. The image of a connected space under a continuous map is ___________.
   a) Connected b) Continuous c) Compact d) Continuum
4. X is normal iff given a closed set A and an open set U containing A, there is an open set V containing A such that _____________
   a) $V \subset U$ b) $\bar{V} \subset U$ c) $\bar{V} = U$ d) $V = U$
5. A subset A of a space X equals the intersection of a countable collection of open subsets of X, then X is called _________
   a) Compactification b) Equivalent c) G_δ set d) $\mathcal{B}$ set

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define: Order Topology.
7. State the Pasting Lemma.
8. Define: Linear Continuum.
9. State Tietze Extension Theorem.
10. When do we say two compactifications Y_1 *and* Y_2 on X are equivalent?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that the topologies of $\mathbb{R}_l$ *and* $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

(OR)

b) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

12. a) Let $f: A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Then prove that f is continuous iff the functions $f_1 : A \to X$ *and* $f_2 : A \to Y$ are continuous.

(OR)

b) State and prove Uniform Limit Theorem.

(CONTD……..2)

13.a) Prove that a finite Cartesian product of connected spaces is connected.

(OR)

b) Prove that every compact subspace of a Hausdorff space is closed.

14. a) Prove that every well – ordered set X is normal in the order topology.

(OR)

b) Prove that a subspace of a completely regular space is completely regular. Also prove that a product of completely regular spaces is completely regular.

15. a) State and prove Tychonff Theorem.

(OR)

b) Let X be a regular space with a basis $\mathcal{B}$ that is countably locally finite. Then prove that X is normal, and every closed set in X is a G_δ set in X.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Let X be a topological space. Then prove the following:

i) $\emptyset$ and X are closed.

ii) Arbitrary intersections of closed sets are closed.

iii) Finite unions of closed sets are closed.

(OR)

b) Let X be a space satisfying the T_1 axiom; Let A be a subset of X. Then prove that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.

17.a) Let X and Y be a topological spaces; let $f: X \to Y$. Then prove that the following are equivalent:

i) f is continuous.
ii) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.
iii) For every closed ser B of Y, the set $f^{-1}(B)$ is closed in X.
iv) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$

(OR)

b) Let $d(a, b) = min\{|a - b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points on $\mathbb{R}^\omega$, define $D(x, y) = \sup\left\{\frac{\bar{d}(x_i,y_i)}{i}\right\}$. Then prove that D is a metric that induces the product topology on $\mathbb{R}^\omega$.

18. a) If L is a Linear Continuum in the order topology, then prove that L is connected and so are intervals and rays in L.

(OR)

b) Prove that the product of finitely many compact spaces is compact.

19. a) i) Prove that a subspace of a Hausdorff space is Hausdorff; a product of Hausdorff spaces is Hausdorff.

ii) Prove that a subspace of a regular space is regular; a product of regular spaces is regular.

(OR)

b) Prove that every regular space with a countable basis is normal.

20.a) State and prove Nagata – Smirnov Metrization Theorem.

(OR)

b) State and prove Smirnov Metrization Theorem.

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