(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2024ONLY)

25PMS104

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS :NOV-2025
M.Sc -MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME : 3 HOURS**

ORDINARY DIFFERENTIAL EQUATIONS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If $y_1(x)$ and $y_2(x)$ are linearly dependent, then their Wronskian $W(y_1, y_2)$ *is*_____.

   a) Zero everywhere b) Non-Zero everywhere

   c) Zero at isolated points only d) Constant Non-Zero

2. The solution of the Legendre equation that is finite at $x = \pm 1$ is called ___.

   a) Legendre polynomial b) Bessel function

   c)Chebyshev polynomial d) Hermite polynomial

3. The matrix exponential e^{At} is used in solving ____.

   a) Non-linear systems only b) Linear systems with constant coefficients

   c) Legendre equations d) Bessel equations

4.The successive approximation method in Picard's theorem constructs solutions in the form ___.

   a) Power series in x b) Iterative integrals

   c) Fourier series d) Matrix exponentials

5.Green's function is primarily used to _____.

   a) Convert a boundary value problem into an integral equation

   b) Solve non-homogeneous systems by eigenvalue method

   c) Find the power series expansions

   d) Eliminate boundary conditions

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

(Qn. No. 6 - 10)

6.What does a zero Wronskian indicate?

(CONTD …. 2)

7.What is an ordinary point of a differential equation?

8.What is the fundamental matrix of a system?

9.What is meant by elementary linear oscillations?

10.Define periodic boundary conditions.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(Qn. No. 11 to 15)Questions for Short Answers with internal choices – 2 questions from each unit. **(K3)**

11. a)State and prove Abel's formula.

(OR)

b) Find the general solution of the equation $x'' - \frac{2}{t^2}x = 0, 0 < t < \infty$.

12. a)If p_n is a Legendre polynomial, prove that $\int_{-1}^{1} {p_n}^2(t)\, dt = \frac{2}{2n+1}$.

(OR)

b)Show that the individual equation for the equation $tx'' + (1-t)x' + nx = 0$ has repeated roots.

13. a)Write the equation $x''' - 6x'' + 11x' - 6x = 0$ in the vector matrix form.

(OR)

b)Find a fundamental matrix for the system $\bar{x}' = A\bar{x}$ where $A = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$

are constants.

14. a)Let x(t) be a solution of $x'' + a(t)x = 0, t \geq 0$ existing on $(0, \infty)$. If a(t) < 0 on $(0, \infty)$, prove that x(t) has atmost one zero.

(OR)

b)Prove that the zeros of a solutions of $x'' + a(t)x' + b(t)x = 0, t \geq 0$, are isolated.

15. a)State and prove Eigen function expansion

(OR)

b)Find the Green's function for the BVP $x'' = f(t); x(0) = x(1) = 0$.

(CONTD …. 3)

SECTION – C **(5 X 8 = 40 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(Qn. No. 16 to 20Questions for Long Answers with internal choices – 2 questions from each unit. **(K4 (Or) K5)**

16. a)Assume that $a_0(t), a_1(t), \dots, a_n(t)$ and b(t) are real valued continuous functions of t defined on an interval I of the real line R and that $a_0(t) \neq 0$,for all t∈I.Prove that the IVP $a_0(t)x^n + a_1(t)x^{(n-1)} + \cdots + a_n(t)x = b(t), x(t_0) = \alpha_1, x'^{(t_0)} = \alpha_2, \dots, x^{(n-1)}(t_0) = \alpha_n$ has one and only one solution.

(OR)

b)Let $x_1(t), x_2(t), \dots, x_n(t)$ be linearly independent solutions of L(x) = 0 on the interval I. prove that any solution x(t) of L(x) = 0 on I is of the form $x(t) = c_1x_1(t) + c_2x_2(t) + \cdots + c_nx_n(t), t\epsilon I$ Where $c_1, c_2, \dots, c_n$ are some constants.

17. a)Consider the Hermite equations$x'' - 2tx' + 2x = 0$. Assume that z(t)=$\sum_{k=0}^{\infty} a_k t^k$ is a solution of Hermite equation. Determine the constantsa_k.

(OR)

b)Let A_1, A_2, ... be the positive zeros of the Bessel function $J_p(t)$, prove that

$$\int_0^1 t\, J_p(A_m t) J_p t(A_n t) dt = \begin{cases} 0 \ if \ m \neq n \\ \frac{1}{2} J_{p+1}(A_n)^2 \ if \ m = n \end{cases}$$

18. a)State and prove existence and uniqueness theorem.

(OR)

b)Determine $exp(tA)$ for the system $x' = Ax$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$

19. a)State and prove Picard's theorem.

(OR)

b)State and prove Hille-winter theorem.

20. a)State and prove Picard's theorem on Boundary value problem.

(OR)

b)Let G(t,s) be given by the relation

$$\mathrm{G(t,s)} = \begin{cases} -(y(t)z(s) \ / \ A \ \ if \ t \leq s \\ -y(s)z(t) \ / \ A \ \ if \ t \geq s \end{cases}$$

prove that x(t) is a solutionof L(x)+f(t)=0,a$\leq t \leq b, m_1x(a) + m_2x'(a) = 0, m_3x(b) + m_4x'(b) = 0$ if and only if $x(t) = \int_a^b G(t,s)\, f(s)$ds.

****