(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED

25PMS103

DURING THE ACADEMIC YEAR 2025 ONLY)

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025
M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME : 3 HOURS**

COMPLEX ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. The poles of the function $\frac{1}{z^2+5z+6}$ are _____.

   a) 1,2 b) 2,3 c) 3,4 d) 4,5

2. A non-constant _____ function has neither a maximum nor minimum in its region.

   a) analytic b) continuous c) local d) harmonic

3. The value of $\Gamma\left(\frac{1}{2}\right)$ is _____.

   a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\pi}{2}$ d) π

4. For genus and order of the entire function, which of the following is true?

   a) $h-1 \leq \lambda \leq h+1$ b) $h \leq \lambda \leq h+1$ c) $h-1 \leq \lambda \leq h$ d) $h = \lambda$

5. If $\omega_1$and ω_2 are periods, then which of the following is not a period?

   a) $\omega_1 + \omega_2$ b) $\omega_1 - \omega_2$ c) $\omega_1\omega_2$ d) $\frac{\omega_1}{\omega_2}$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. When will you call a chain as cycle?
7. Write down the Laplace's equation.
8. Write down the Taylor's expansion of an analytic function $f(z)$ in a region containingz_0.
9. What can we say about the family of analytic functions with positive real part?
10. What do you mean by periodic function?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points which do not belong to Ω.

(OR)

b) Find $\int_0^{\pi} \log\sin x \, dx$.

12. a) If u_1 and u_2 are harmonic in Ω, then show that $\int u_1 * d\, u_2 - u_2 * d\, u_1 = 0$ for every cycle γ which is homologous to zero in Ω.

(OR)

b) Discuss the geometric interpretation of Poisson's formula. **(CONTD……2)**

13.a) If the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω and if $f_n(z)$ converges to $f(z)$ uniformly on every compact subset of Ω, then prove that $f(z)$ is either identically zero or never equal to zero in Ω.

(OR)

b) Prove the identity $\frac{\pi^2}{\sin^2 \pi z} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

14. a) Show that the family $\mathfrak{F}$ is normal if and only if its closure $\mathfrak{F}^-$ with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \delta_g(f,g)2^{-k}$ is compact.

(OR)

b) Derive the zeros of Zeta function.

15.a) Show that a discrete module consists of either of zero alone of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$ or of all linear combinations $n_1\omega_1 + n_2\omega_2$ with integral coefficients of two numbers ω_1, ω_2 with non real ratio $\frac{\omega_2}{\omega_1}$.

(OR)

b) Show that a non constant elliptic function has equally many poles as it has zeros.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Examine the statement that "if $f(z)$ is analytic in Ω, then $\int f(z)dz = 0$ for every cycle γ which is homologous to zero in Ω".

(OR)

b) Interpret the Cauchy's residue theorem by writing the definition of Residues.

17. a) State and prove the Schwarz's theorem.

(OR)

b) Let Ω^+ be the part in the upper half plane of a symmetric region Ω and let σ be the part of the real axis in Ω. Suppose that $v(x)$ is continuous in $\Omega^+ \cup \sigma$, a harmonic in Ω^+ and zero on σ. Then prove that v has a harmonic extension Ω that satisfies the symmetric relation $v(\bar{z}) = -v(z)$.

18.a) State and prove Weierstrass theorem.

(OR)

b) Formulate the Legendre's duplication formula.

19.a) Derive Jensen's formula.

(OR)

b) Prove that the family $\mathfrak{F}$ is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\epsilon > 0$, it is possible to find $f_1, f_2, \ldots, f_n \in \mathfrak{F}$ such that for every $f \in \mathfrak{F}$ satisfies $d(f, f_i) < \epsilon$ for some f_i.

20.a) Prove that any two bases of the same module are connected by a unimodular transformation.

(OR)

b) Prove that the zeros $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy $a_1 + a_2 + \ldots + a_n \equiv b_1 + b_2 + \ldots + b_n \pmod{n}$.

****