

(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2025 ONLY)

25PMS102

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI
END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2025
M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME: 3 HOURS**

25PMS102 - REAL ANALYSIS

SECTION – A

(10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS.

(K1)

1. If $f \in \mathfrak{R}(\alpha)$ and $g \in \mathfrak{R}(\alpha)$ on $[a, b]$, then which one of the following is true?
(a) $fg \notin \mathfrak{R}(\alpha)$ (b) $|f| \notin \mathfrak{R}(\alpha)$ (c) $fg \in \mathfrak{R}(\alpha)$ (d) $\left| \int_a^b f \, d\alpha \right| \geq \int_a^b |f| \, d\alpha$
2. The value of $\lim_{n \rightarrow \infty} \frac{n}{2n+2}$ is _____.
(a) 1 (b) ∞ (c) 0 (d) $\frac{1}{2}$
3. A linear operator A on \mathcal{R}^n is invertible then $\det[A]$ is _____.
(a) $\neq 0$ (b) = 0 (c) $\neq 1$ (d) $\neq \infty$
4. The validity of the following statements:_____.
Statement A: The Lebesgue measure of an interval is its length
Statement B : Lebesgue measure is translation variant
(a) only A is true (b) both A and B are true
(c) only B is true (d) both A and B are false
5. A measurable function f on E is said to be integrable over E provided $|f|$ is integrable over E .
The integral $\int_E f =$ _____.
(a) $\int_E f^+ + \int_E f^-$ (b) $\int_E f^+ - \int_E f^-$ (c) $\int_E f^- - \int_E f^+$ (d) both (b) and (c)

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

6. Write the Riemann-Stieltjes integral for the function f with respect to α .
7. Estimate the value of $f'_n(0)$ as $n \rightarrow \infty$, if $f_n(x) = \frac{\sin x}{\sqrt{n}}$.
8. Define independent vectors.
9. Define characteristic function of any set A .
10. Write the lower and upper Riemann integrals.

SECTION – B

(5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If $f \in \mathfrak{R}$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$
then find the value of $\int_a^b f(x) \, dx$.

(OR)

- b) If P^* is a refinement of P , then identify the relation between $L(P, f, \alpha)$ and $L(P^*, f, \alpha)$.

(CONTD....2)

12. a) Consider the sequence $s_{m,n} = \frac{m}{m+n}$, for $m = 1,2,3, \dots$ and $n = 1,2,3, \dots$. Compute the value of $\lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} s_{m,n}$ and $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} s_{m,n}$.
- (OR)**
- b) If $f_n \rightarrow f$ uniformly on a set E in a metric space, x is a limit point of E and suppose that $\lim_{t \rightarrow x} f_n(t) = A_n$, then show that $\{A_n\}$ converges and $\lim_{t \rightarrow x} f(t) = \lim_{n \rightarrow \infty} A_n$.
- 13.a) If a linear operator A on a finite dimensional vector space X is one-to-one then determine the range of A .
- (OR)**
- b) If $[A]$ and $[B]$ are n by n matrices, then show that $\det([B][A]) = \det[B]\det[A]$.
14. a) If f and g are measurable functions on E that are finite almost everywhere on E . For any α, β , identify the measurability of $\alpha f + \beta g$.
- (OR)**
- b) Prove that the union of a finite collection of measurable sets is measurable.
15. a) If f is a bounded measurable function on a set of finite measure E . Show that f is integrable over E .
- (OR)**
- b) Compute the measure of $\{x \in E \mid f(x) \geq \lambda\}$, where $\lambda > 0$ and f is a nonnegative measurable function on E .

SECTION – C**(5 X 8 = 40 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.****(K4 (Or) K5)**

16. a) Prove that $f \in \mathfrak{R}(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.
- (OR)**
- b) If $f \in \mathfrak{R}(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$ then prove that $h \in \mathfrak{R}(\alpha)$ on $[a, b]$.
17. a) There exists a real continuous function on the real line which is nowhere differentiable. Justify this statement with suitable example.
- (OR)**
- b) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3, \dots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K , then prove the following:
- (i) $\{f_n\}$ is uniformly bounded on K
- (ii) $\{f_n\}$ contains a uniformly convergent subsequence.
18. a) Consider Ω is the set of all invertible linear operators on \mathcal{R}^n . Prove the following:
- (i) If $A \in \Omega$, $B \in L(\mathcal{R}^n)$ and $\|B - A\| \cdot \|A^{-1}\| < 1$ then $B \in \Omega$.
- (ii) Ω is an open subset of $L(\mathcal{R}^n)$ and the mapping $A \rightarrow A^{-1}$ is continuous on Ω .
- (OR)**
- b) Define contraction mapping. Prove that in a complete metric space X , there exists one and only one $x \in X$ such that $\phi(x) = x$, where ϕ is a contraction of X into X .
19. a) Prove that the disprove: The outer measure of an interval is its length.
- (OR)**
- b) State and prove the Simple approximation theorem.
20. a) State and prove the Bounded Convergence theorem.
- (OR)**
- b) State and prove Fatou's Lemma.