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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2025 ONLY)

25PMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2025
M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME: 3 HOURS**

25PMS101 – ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. If A, B are finite subgroups of G, then $o(AxB) =$_____.

   (a) $\frac{o(A)o(B)}{o(A\cap xBx)}$ (b) $\frac{o(A)o(B)}{o(A\cup xBx^{-1})}$ (c) $\frac{o(A)o(B)}{o(A\cap xBx^{-1})}$ (d) $\frac{o(A\cap B)o(B)}{o(A\cap xBx^{-1})}$

2. If $f(x), g(x)$ are two nonzero elements of $F[x]$, then which one of the following is true?

   (a) deg $\big(f(x)g(x)\big) \leq$ deg $f(x)$ + deg $g(x)$ (b) deg $\big(f(x)g(x)\big) =$ deg $f(x)$ + deg $g(x)$

   (c) deg $\big(f(x)\big) \geq$ deg $f(x)g(x)$ (d) deg $\big(f(x)\big) \geq$ deg $(f(x) + g(x))$

3. A complex number is said to be an algebraic number if it is algebraic over the field of ____.

   (a) Integers (b) rational numbers (c) real numbers (d) natural numbers

4. Every _____group G is a module over the ring of integers.

   (a) abelian (b) cyclic (c) non-abelian (d) non – cyclic

5. If α is constructible then α lies in some extension of the rationals of degree a power of _____.

   (a) 0 (b) 1 (c) 3 (d) 2

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define the center of a group G.
7. State the Pigeonhole principle.
8. Write two finite fields having the same number of elements ?
9. Define orthonormal set.
10. When do we say that a real number is to be a constructible number?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Show that the normalizer of 'a' in G, $N(a)$ is a subgroup of G.

(OR)

b) How many 11 – Sylow subgroups and 13 – Sylow subgroups of a group G of order $11^2.13^2$?

(CONTD …. 2)

12.a) State and prove the division algorithm.

(OR)

b) Show that a finite integral domain is a field.

13. a) Show that the multiplicative group of nonzero elements of a finite field is cyclic.

(OR)

b) If L is an algebraic extension of K and if K is an algebraic extension of F then prove that L is an algebraic extension of F.

14. a) If V is a finite-dimensional inner product space and W is a subspace of V then identify the value of $(W^{\perp})^{\perp}$.

(OR)

b) Determine the relation between $|(u, v)|$ and $\|u\|, \|v\|$, where $u, v \in V$, a vector space.

15. a) By straightedge and compass it is impossible to trisect $60°$. Justify.

(OR)

b) Prove that the regular hexagon is constructible.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Prove that the number of $p-$ Sylow subgroups in a group G for a given prime, is of the form $1 + kp$.

(OR)

b) If p is a prime number and $p|o(G)$ then prove that G has an element of order p.

17.a) Discuss the Eisenstein criterion.

(OR)

b) If F is a field, then prove that $F[x_1, x_2, \ldots, x_n]$ is a unique factorization domain.

18. a) If the finite field F has p^m elements then prove that every $a \in F$ satisfies $a^{p^m} = a$.

(OR)

b) If L is a finite extension of K and if K is a finite extension of F then show that L is a finite extension of F.

19. a) Prove or disprove : The number ‘e’ is transcendental.

(OR)

b) Prove that V has an orthonormal set as a basis if V is a finite-dimensional inner product space.

20.a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root iff $f(x)$ and $f'(x)$ have nontrivial common factor.

(OR)

b) If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

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