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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS513

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS :NOVEMBER-2025
B.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: V **TIME : 3 HOURS**

PART – III
COMPLEX ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. The value of $i^2 =$_____.

a) -1 b) 1 c) i d) 0

2. The value of $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)x =$_____.

a) 1 b) 0 c)x d) $+\infty$

3. The value of $e^{iz} =$_____.

a) $\cos z - i\sin z$ b) $\cos z + i\sin z$ c) $\cos z$ d) $\sin z$

4. If $f(z)$ is analytic in an open disk Δ, then $\int_v f(z)dz =$___, for every closed curve v in Δ.____

a) 0 b) πi c) $2\pi i$ d) $-\pi i$

5. If $\lim_{z\to a} f(z) = \infty$, the point a is said to be a _____ of $f(z)$

a) zero b) pole c) essential singularity d) none

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. If $a = \alpha + i\beta$, then find $a\bar{a}$.
7. Define power series.
8. State addition theorem for exponential.
9. Compute $\int_{|z|=1} \frac{e^z}{z} dz$
10. Define meromorphic function.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11.a) Find the value of $\frac{5}{-3+4i}$

(OR)

b) Show that $|ab| = |a|.|b|$, where ' a ' is a complex number.

12.a) Define (i) Cauchy sequence (ii) Uniform convergence.

(OR)

b) Prove that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.

13.a) Write a note on trigonometric functions.

(OR)

b) Discuss arcs and closed curves.

(CONTD…….2)

14.a)If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of $2\pi i$.

(OR)

b)Show that $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|dt$

15.a)Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

(OR)

b)State and prove Schwarz Lemma.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4& K5)

16. a)Find the absolute value of $\frac{(3+4i)(-1+2i)}{(-1-i)(3-i)}$ **(K4)**

(OR)

b)Find the value of $\left(\frac{2+i}{3-2i}\right)^2$

17.a) State and prove Abel's limit theorem. **(K4)**

(OR)

b) Prove that for every power series, there exists a number $R, 0 \leq R \leq \infty$, called the radius of convergence with the following properties:

(i) The series converges absolutely for every z with $|z| < R$. If $0 \leq P < R$ the convergence is uniform for $|z| \leqslant \rho$.

(ii) If $|z| > R$ the terms of the series are unbounded and the series is consequently divergent. **(K4)**

18. a) Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant. The same is true if either the real part or the imaginary part is constant. **(K4)**

(OR)

b) If $f(z)$ is analytic in Ω and $f'(z) \neq 0$, then prove that $\omega = f(z)$ is conformal. **(K4)**

19.a) State and prove Cauchy's theorem for a rectangle. **(K5)**

(OR)

b) Suppose that $f(z)$ is analytic in an open disk $\triangle$ and let γ be a closed curve in Δ. For any point ' a ' not on γ, prove that $n(\gamma, a) \cdot f(a) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)dz}{z-a}$, where $n(\gamma, a)$ is the index of a with respect to γ. **(K5)**

20.a) State and prove Taylor's theorem. **(K5)**

(OR)

b) State and prove Maximum principle. **(K5)**

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