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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS511

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025

B.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: V **TIME : 3 HOURS**

PART - III

REAL ANALYSIS - I

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS. **(K1)**
MULTIPLE CHOICE QUESTIONS.

1. If $x > 0$ and $y > 0$ then ________.
   a) $xy > 0$ b) $xy = 0$ c) $x = 0$ d) $xy < 0$
2. Any set of ordered pairs is called a _________ .
   a) domain   b) bijection   c) function   d) relation
3. If $B - A = B$ if $B \cap A$ is ___________ .
   a) A   b) B   c) empty   d) non-empty
4. The set of numbers of the form $1/n$, $n = 1, 2, 3, \ldots$ has ___________ as an accumulation point.
   a) 0   b) 1   c) infinity   d) 2
5. Which of the following set is complete?
   a) $(0,1]$ b) $[0, 1]$ c) $(0, 1)$ d) $(0, 2]$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. If $S = [0, 1)$, what is the maximum element and least upper bound of S?
7. Define the cartesian product of two sets.
8. Define the norm.
9. Write the definition of an accumulation point.
10. Define a Complete Metric Space.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If $a|bc$ and $(a, b) = 1$, then prove $a|c$.

(OR)

b) Prove that the set Z^+ of positive integers 1, 2, 3, ... is unbounded above.

12. a) Let Z^+ denote the set of all positive integers. Then show that the cartesian product $Z^+ \times Z^+$ is countable

(OR)

b) Prove that every subset of a countable set is countable.

13.a) Prove that the set Q of all rational numbers is a countable set.

(OR)

b) Show that the set S of intervals with rational endpoints is a countable set.

14.a) Let F be an open covering of a closed and bounded set A in R". Then show that a finite subcollection of F also covers A.

(OR)

b) Let (S, d) be a metric subspace of (M, d) and let Y be a subset of S. Then prove that Y is closed in S if, and only if, Y = B ∩ S for some set B which is closed in M.

15.a) Let X be a closed subset of a compact metric space M. Then show that X is compact.

(OR)

b) Prove that in a metric space (S, d) a sequence $\{x_n\}$ converges to p if, and only if, every subsequence $\{x_{k(n)}\}$ converges to p.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Show that every pair of integers a and b has a common divisor d of the form d = ax + by where x and y are integers. Moreover, every common divisor of a and b divides this d.

(OR)

b) State and prove Unique factorization theorem.

17.a) Prove that the set of all real numbers is uncountable.

(OR)

b) State and prove Cauchy-Schwarz inequality.

18. a) Show that the intersection of a finite collection of open sets is open.

(OR)

b) Prove that every non empty open set S in R^1 is the union of a countable collection of disjoint component intervals of S.

19.a) Prove that a set S in R^n is closed if, and only if, it contains all its adherent points.

(OR)

b) State and prove Bolzano-Weierstrass theorem.

20.a) In a metric space (S, d), assume $x_n \to p$ and let T = $\{x_1, x_2, \ldots\}$ be the range of $\{x_n\}$. Then prove that

i) T is bounded.

ii) p is an adherent point of T

(OR)

b) In any metric space (S, d), prove that every compact subset T is complete.

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