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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS510

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025

B.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: V **TIME : 3 HOURS**

PART – III
MODERN ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

MULTIPLE CHOICE QUESTIONS.

1. Let G consist of the real numbers 1, - 1 under the multiplication of real numbers. G is then an abelian group of order _____.
   a) 1 b) 2 c) 0 d) 3
2. Every subgroup of an abelian group is _____.
   a) normal b) bijection c) cyclic d) prime
3. Every permutation is a product of _____.
   a) 1 cycle b) 4 cycles c) 3 cycles d) 2 cycles
4. A _____ is a commutative division ring.
   a) field b) Integral domain c) ring d) group
5. Every _____ can be imbedded in a field.
   a) normalizer b) integral domain c) ring d) cyclic group

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define equivalent points.
7. Let G be the group of integers under addition defined by $\phi(x) = 2x$ where $x \epsilon G$. Prove that ϕ is a homomorphism.
8. What is the product of two odd permutations?
9. Define a division ring.
10. When is a/b = c/d?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If $\sigma: S \to T, \tau: T \to \cup$ and $\mu: \cup \to V$ then prove that $(\sigma \circ \tau) \circ \mu - \sigma \circ (\tau \circ \mu)$

(OR)

b) Prove that a nonempty subset H of the group G is a subgroup of G if and only if
   (i) a, b ϵ H implies that ab ϵ H.
   (ii) a ϵ H implies that $a^{-1} \epsilon$ H.

12. a) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \epsilon G$.

(OR)

b) Suppose G is a group, N a normal subgroup of G; define the mapping ϕ from G to G/N by $\phi(x) = Nx$ for all $x \epsilon G$. Then prove that ϕ is a homomorphism of G onto G/N.

(CONTD…….2)

13.a) Prove that every permutation is the product of its cycles.

(OR)

b) Find the orbits and cycles of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$$

14. a) If p is a prime number, then show that J_p, the ring of integers mod p, is a field.

(OR)

b) If U is an ideal of R and $1 \in U$, then prove that U = R.

15. a) Prove that the mapping $\phi: D \rightarrow F$ defined by $\phi(a) = [a, 1]$ is an isomorphism of D into F.

(OR)

b) Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists an element $a_0 \in A$ such that A consists exactly of all a_0x as x ranges over R.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) If G is a group, then prove that
(i) The identity element of G is unique.
(ii) Every $a \in G$ has a unique inverse in G.
(iii) For every $a \in G$, $(a^{-1})^{-1} = a$.
(iv) For all $a, b \in G$, $(a \cdot b)^{-1} = b^{-1}.a^{-1}$

(OR)

b) If H and K are finite subgroups of G of orders o(H) and o(K), respectively, then prove that

$$o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$$

17. a) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

(OR)

b) Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Then prove that $G/K \approx \bar{G}$.

18. a) Show that every group is isomorphic to a subgroup of A(S) for some appropriate S.

(OR)

b) If G is a finite group, then prove that $c_a = o(G)/o(N(a))$; in other words, the number of elements conjugate to a in G is the index of the normalizer of a in G.

19. a) If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.

(OR)

b) Prove that a finite integral domain is a field.

20. a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

(OR)

b) If p is a prime number of the form 4n + 1, then prove that $p = a^2 + b^2$ for some integers a and b.

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