**(FOR THE CANDIDATES ADMITTED**

**DURING THE ACADEMIC YEAR 2025 ONLY)**

**25UMS101**

**REG.NO.:**

**N.G.M. COLLEGE (AUTONOMOUS): POLLACHI**
**END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2025**
**B.Sc.-MATHEMATICS** **MAXIMUM MARKS: 75**
**SEMESTER: I** **TIME: 3 HOURS**

## PART-III
## CLASSICAL ALGEBRA

**SECTION – A** **(10 X 1 = 10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS.**
**MULTIPLE CHOICE QUESTIONS.** **(K1)**

1) If $u_1 + u_2 + \cdots + u_n + \cdots$ is _____, then $\lim_{n\to\infty} u_n = 0$.
   (A) convergent (B) divergent
   (C) oscillates finitely (D) oscillates infinitely

2) $1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots =$ _____.
   (A) $(1+x)^{-n}$ (B)$(1-x)^{-n}$ (C)$(1-x)^{n}$ (D)$(1-x)^{-n}$

3) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 + qx + r = 0$, then $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta) =$ _____.
   (A) $-qr$ (B) $qr$ (C) r (D) -r

4) The equation$x^3 - 3x + 1 = 0$ has a root between _____.
   (A) 0 and 1 (B) 2 and 3 (C) $-1$ and 0 (D)3 and 4

5) If the eigen values of an orthogonal matrix A are positive,then $|A| =$ _____.
   (A) Sum of the eigen values (B)0 (C)1 (D) -1

**ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES.** **(K2)**

6) State Cauchy's root test.
7) Find the value of $log\left(\frac{1+x}{1-x}\right)$.
8) Find the quotient and remainder when $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$.
9) State a rule for finding the multiple roots of an equation $f(x) = 0$.
10) Prove that $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$ is idempotent.

**SECTION – B** **(5 X 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.** **(K3)**

11) (a) Prove that the series $\frac{1}{1\cdot3} + \frac{2}{3\cdot5} + \frac{3}{5\cdot7} + \cdots$ is divergent.

**(OR)**

(b) Test the convergency of the series $\sum_{n=0}^{\infty} \frac{n^3+1}{2^n+1}$.

**(CONTD……2)**

12) (a) Sum the series $\frac{1^2}{1!}+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+\cdots+\frac{1^2+2^2+3^2+\cdots+n^2}{n!}+\cdots$

**(OR)**

(b) Show that $\frac{5}{1\cdot2\cdot3}+\frac{7}{3\cdot4\cdot5}+\frac{9}{5\cdot6\cdot7}+\cdots\infty=3\log 2-1.$

13) (a) If $a+b+c+d=0$, show that $\frac{a^5+b^5+c^5+d^5}{5}=\frac{a^2+b^2+c^2+d^2}{2}\cdot\frac{a^3+b^3+c^3+d^3}{3}$.

**(OR)**

(b) Increase by 7 the roots of the equation $3x^4+7x^3-15x^2+x-2=0$.

14) (a) Find the condition that the cubic equation $ax^3-3bx^2+3cx+d=0$ has two equal roots and when the condition is satisfied, find the equal roots.

**(OR)**

(b) Find the number of real roots of the equation $x^4-14x^2+16x+9=0$ by using Strum's functions.

15) (a) Define orthogonal matrices. If A and B are orthogonal, then prove that AB is orthogonal.

**(OR)**

(b) Using Cayley-Hamilton theorem, find $A^{-1}$if $A=\begin{bmatrix}1&2&-2\\2&5&-4\\3&7&-5\end{bmatrix}$. Also verify the theorem.

**SECTION – C (5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)**

16) (a) Show that the series $\frac{1}{1^k}+\frac{1}{2^k}+\frac{1}{3^k}+\cdots$ is convergent when k is greater than unity and divergent when k is equal to or less than unity.

**(OR)**

(b) Test the convergency and divergency of the series $1+\frac{2x}{2!}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+\cdots$

17) (a) Sum the series to infinity $\frac{15}{16}+\frac{15\cdot21}{16\cdot24}+\frac{15\cdot21\cdot27}{16\cdot24\cdot32}+\cdots$

**(OR)**

(b) State and prove the Exponential theorem.

18) a) If $\alpha,\beta,\gamma$ are the roots of the equation $x^3+px^2+qx+r=0$, from the equation whose roots are$\beta+\gamma-2\alpha,\gamma+\alpha-2\beta,\alpha+\beta-2\gamma$ .

**(OR)**

(b) Slove the equation $x^4+20x^3+143x^2+430x+462=0$ by removing its second term.

19) a) Find all the rational roots of the equation $4x^3+20x^2-23x+6=0$.

**(OR)**

(b) Find the positive root of the equation $x^3-2x^2-3x-4=0$ correct to three places of decimals.

20) (a) Find the eigen values and eigen vectors of $A=\begin{bmatrix}2&2&0\\2&1&1\\-7&2&-3\end{bmatrix}$.

**(OR)**

(b) Diagonalise the matrix $A=\begin{bmatrix}3&-1&1\\-1&5&-1\\1&-1&3\end{bmatrix}$ and hence find $A^4$.

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