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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2025 ONLY)

25UMS1A1

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2025

B.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

PART - III

MATHEMATICAL STATISTICS-I

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. The marginal probability functions of Y and X are given respectively by $f_Y(y)$ and $f_X(x)$ where $f_Y(y) =$ __________ for continuous variables.

   (a) $\int_{-\infty}^{\infty} f_{XY}(x,y)dx$ (b) $\int_{-\infty}^{\infty} f_{XY}(x,y)dy$ (c) $\sum_X \rho_{XY}(x,y)$ (d) $\sum_Y \rho_{XY}(x,y)$

2. If X and Y are independent then cov (X,Y)=______

   (a) 1 (b) 0 (c) ∞ (d) cov(Y,X)

3. The probability distribution of the number of success so obtained is called _________.

   (a) Poisson (b) Gamma (c) Binomial distribution (d) Normal

4. In the normal distribution ___ are coincide

   (a) Mode and median (b) mean, median and mode
   (c) Mean and standard deviation (d) mean and variance

5. ______ is the variance of Exponential distribution.

   (a) $\frac{1}{\theta}$ (b) $\frac{1}{\theta^2}$ (c) λ (d) λ^2

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define called stochastically independent?
7. If C is constant, then show that $Var(CX) = C^2 Var(X)$
8. Write the mean and variance of Poisson distribution.
9. Write the MGF of Rectangular distribution.
10. Define Gamma distribution.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11.a) The joint p.d.f of two random variables X and Y is given by

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^2(1+y)^2}; \begin{pmatrix} 0 < x < \infty \\ 0 < y < \infty \end{pmatrix}$$

Applying the basic concepts, find the marginal distribution of X and Y and the conditional distribution of Y for X=x.

(OR)

b) If X and Y are two random variable having joint density function

$$f(x,y) = \begin{cases} \frac{1}{2}(8-x-y) & 0 < x < 2; 2 < y < 4 \\ 0 & otherwise \end{cases}$$

Calculate (i) $P(X < 1 \cap Y < 3)$ (ii) $P(X+Y < 3)$ and (iii) $P(X < 1 | Y < 3)$

(CONTD….2)

12.a) Let $X_1, X_2, X_3,......................X_n$ be a random variables then prove that

$$V\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{i<j}\sum_{j=1}^{n} a_i a_j Cov(X_i, X_j)$$

(OR)

b) Find the expectation of the number on a die when thrown. (b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

13.a) State and prove Chebychev's inequality

(OR)

b) Discuss recurrence relation for the moments of Binomial distribution

14.a) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. Assuming the above distribution, Evaluate the mean and standard deviation of the distribution?

(OR)

b) Show that for the rectangular distribution $f(x) = \frac{1}{2a},\ -a < x < a$ the MGF about origin is $\frac{1}{at}\sinh at$. Also show that moments of even order are given by $\mu_{2n} = \frac{a^{2n}}{(2n+1)}$

15. a) State and prove the additive property of Gamma distribution.

(OR)

b) Derive the characteristic function of Gamma distribution.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16.a) For the following bivariate probability distribution of X and Y Determine (i) $P(X < 1, Y = 2)$ (ii) $P(X < 1)$ (iii) $P(Y = 3)$ (iv) $P(Y \le 3)$ and (v) $P(X < 3, Y < 4)$

X↓ Y→	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0`	$\frac{2}{64}$

(OR)

b) The joint probability density function of two random variable X and Y is

$$f(x,y) = \begin{cases} kx(x-y)\ , & 0 < x < 2; -x < y < x \\ 0\ , & otherwise \end{cases}$$

Evaluate the constant k and obtain the marginal distribution of X & Y and the conditional distribution of Y for X=x given.

17.a) Three urns contain respectively 3 green and 2 white balls , 5 green and 6 white balls and 2 green and 4 white balls. One ball is drawn from each urn. Calculate the expected number of white balls drawn out.

(OR)

(CONTD….3)

17b) Two random variables X and Y have the following joint probability density function:

$$f(x,y)=\begin{cases}2-x-y; & 0\le x\le 1,\ 0\le y\le 1\\ 0 & otherwise\end{cases}$$

Calculate

(i) Marginal probability density function of X and Y

(ii) Conditional density functions

(iii) Var(X) and Var(Y)

(iv) Co-variance between X and Y

18.a) Seven unbiased coins are tossed and number of heads noted. The experiment is repeated 128 timers and the following distribution is obtained:

No of heads	0	1	2	3	4	5	6	7	Total
Frequencies	7	6	19	35	30	23	7	1	128

Reframe the binomial distribution for assuming

(i) The coin is unbiased

(ii) The nature of the coin is not known.

(OR)

b) Fit a Poisson distribution to the following data which gives the number of doddens in a sample of clover seeds.

No of doddens(x)	0	1	2	3	4	5	6	7	8
Observed frequency (f)	56	156	132	92	37	22	4	0	1

19.a) If X is normally distributed and the mean of X is 12 and S.D is 4, find out the probability of the following (i) $X \ge 20$ (ii) $X \le 20$ (iii) $0 \le X \le 12$ (iv) Find x' when $P(X > x') = 0.24$

(OR)

19b) If X is a normal variate with mean 30 and S.D 5. Find the following probabilities (i) $26 \le X \le 40$ (ii) $X \ge 45$ (iii) $|X-30| > 5$

20.a) Derive the constants of Beta distribution of first kind

(OR)

20.b) Derive the constants of Beta distribution of second kind

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ETHICAL PAPER