(NO. OF PAGES: 3)

(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS206

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2025

M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: II **TIME : 3 HOURS**

MATHEMATICAL STATISTICS

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. In the conditional probability of event A, given event B as, _____.

   a) $P\left(\frac{A}{B}\right) = P(A) + P(B)$ b) $P\left(\frac{A}{B}\right) = P(A)/\,P(B)$

   c) $P\left(\frac{A}{B}\right) = P(A \cap B)/P(B)$ d) $P\left(\frac{A}{B}\right) = P(A \cup B)/P(A)$

2. Which of the following distribution is used to compare two variables?

   a) T-distribution b) F- distribution

   c) Normal distribution d) Poisson distribution

3. A point estimator is said to be consistent when _____.

   a) It is smaller standard error

   b) Its expected value is equal to the population parameter

   c) It tends to become closer to the population parameter as the sample size becomes larger.

   d) It is based on all available information.

4. ______ is an unbiased estimator of σ^2.

   a) S b) S^2 c) S_P^2 d) S_σ^P

5. Type I error in statistical hypothesis testing is _____.

   a) Falling to accept the null hypothesis when it is false.

   b) Rejecting the null hypothesis when it is true.

   c) Falling to reject the null hypothesis when it is true

   d) Accepting the null hypothesis when it is false.

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. Define random variable.

7. Define population distribution.

8. Define point estimation.

9. Define confidence interval.

10. What is P-value?

(CONTD………2)

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find a formula for the probability distribution of the total number of heads obtained in four tosses of a balanced coin.

(OR)

b) Given the joint probability density

$$f(x,y) = \begin{cases} 4xy & for\ 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

find the marginal densities of X and Y and the conditional density of X given $Y = y$.

12. a) If X_r and X_s are the rth and sth random variables of a random sample of size n drawn from the finite population $\{c_1, c_2, \ldots, c_N\}$, then prove that $cov(X_r, X_s) = -\frac{\sigma^2}{N-1}$.

(OR)

b) In 16 one-hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a standard deviation of 2.1 gallons. Test the claim that the average gasoline consumption of this engine is 12.0 gallons per hour.

13. a) Show that $\bar{X}$ is a minimum variance unbiased estimator of the mean μ of a normal Population.

(OR)

b) If $x_1, x_2, \ldots x_n$ are the values of a random sample from an exponential population Find the maximum likelihood estimator of its parameter θ.

14.a) A paint manufacturer wants to determine the average drying time of a new interior wall paint. If for 12 test areas of equal size he obtained a mean drying time of 66.3 minutes and a standard deviation of 8.4 minutes, construct a 95% confidence interval for the true mean μ.

(OR)

b) In 16 test runs the gasoline consumption of an experimental engine had a standard deviation of 2.2 gallons. Construct a 99% confidence interval for $\sigma 2$, which measures the true variability of the gasoline consumption of the engine.

15. a) An oil company claims that less than 20 percent of all car owners have not tried its gasoline. Test this claim at the 0.01 level of significance if a random check reveals that 22 of 200 car owners have not tried the oil company's gasoline.

(OR)

b) The specifications for a certain kind of ribbon call for a mean breaking strength of 185 pounds. If five pieces randomly selected from different rolls have breaking strengths of 171.6, 191.8, 178.3, 184.9, and 189.1 pounds, test the null hypothesis $\mu = 185$ pounds against the alternative hypothesis $\mu < 185$ pounds at the 0.05 level of significance.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Find the distribution function of the total number of heads obtained in four tosses of a balanced coin.

(OR)

b) Given the joint probability density

$$f(x,y) = \begin{cases} \frac{2}{3}(x + 2y) & for\ 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases},$$

find the marginal densities of X and Y.

Find the conditional density of X given $Y = y$, and use it to evaluate $P\left(X \le \frac{1}{2} \middle| y = \frac{1}{2}\right)$.

(CONTD………3)

17. a) State and prove Central Limit Theorem.

(OR)

b) If $\bar{X}$ is the mean of a random sample of size *n* taken without replacement from a finite population of size *N* with the mean μ and the variance σ^2, then show that $E(\bar{X}) = \mu$ and $var\ (\bar{X}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$.

18. a) If $X_1, X_2, \dots X_n$ constitute a random sample of size *n* from a Bernoulli population, show that $\widehat{\Theta} = \frac{X_1, X_2, \dots X_n}{n}$ is a sufficient estimator of the parameter θ.

(OR)

b) If $X_1, X_2, \dots X_n$ constitute a random sample of size *n* from a normal population with the mean μ and the varianceσ^2, find joint maximum likelihood estimates of these two parameters.

19. a) If a random sample of size $n = 20$ from a normal population with the variance $\sigma^2 = 225$ has the mean $\bar{x} = 64.3$, construct a 95% confidence interval for the population mean μ.

(OR)

b) If 132 of 200 male voters and 90 of 150 female voters favor a certain candidate running for governor of Illinois, find a 99% confidence interval for the difference between the actual proportions of male and female voters who favor the candidate.

20.a) (i) Suppose that we want to test the null hypothesis that the mean of a normal population with $\sigma^2 = 1$ is μ_0 against the alternative hypothesis that it is μ_1, where $\mu_1 > \mu_0$. Find the value of K such that $\bar{x} > K$ provides a critical region of size α = 0.05 for a random sample of size n.

(ii) With reference to (i), determine the minimum sample size needed to test the null hypothesis $\mu_0 = 10$ against the alternative hypothesis $\mu_1 = 11$ with $\beta \leq 0.06$.

(OR)

b) Suppose that 100 high-performance tires made by a certain manufacturer lasted on the average 21,819 miles with a standard deviation of 1,295 miles. Test the null hypothesis $\mu = 22{,}000$ miles against the alternative hypothesis $\mu < 22{,}000$ miles at the 0.05 level of significance.

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