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(FOR THE CANDIDATES ADMITTED 24UAI1A1

DURING THE ACADEMIC YEAR 2024 ONLY) REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER - 2024

B.Sc (Computer Science with AI & ML) **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

PART - III

24UAI1A1– INTRODUCTION TO LINEAR ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

1. If a system of simultaneous equations has infinite solutions, then that system of equations is called_______.

   (a) Opposite (b) Independent (c) Dependent (d) Same

2. A matrix A with n rows and n columns is called a ______ matrix of order n.

   (a) Square (b) Row (c) Column (d) Diagonal

3. A square matrix whose determinant is equal to zero is called a _______matrix.

   (a) Singular (b) Non – Singular (c) Invertible (d) Transpose

4. The characteristic space of a matrix A corresponding to a characteristic root λ is just the null space of the matrix _______.

   (a) (A-λI) (b) A^{-1} (c) $A^{-1}A$ (d) (A+λI)

5. A set of two vectors $\{V_1 \quad V_2\}$ is linearly dependent if at-least one of the vector is___________.

   (a) a multiple of the other (b) Both vectors are equal

   (c) Both vectors are different (d) Not a multiple of the other

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES

6. What is a system of linear equations?
7. Define matrix.
8. What is the meant by rank matrix?
9. Define characteristic polynomial.
10. What is meant by vector space?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

11. a) Explain Linear dependence and independence.

(OR)

(CONT…2)

Ethical paper

b) Solve the matrix equation Ax = b if A = $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ **, b =** $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$

12. a) Explain the following matrices. i) Square matrix ii) Row matrix iii) Column matrix iv) Zero matrix v) Equal matrices.

(OR)

b) Prove that the matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is unitary

13. a) Find the inverse matrix of A = $\begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$

(OR)

b) Determine the rank of the following matrices: A = $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$.

14. a) Prove that λ is a characteristic root of a matrix A if and only if there exists a non – zero vector X such that AX = λX .

(OR)

b) Determine the characteristic roots of a matrix A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$.

15. a) Show that $v_1, v_2, v_3, \ldots v_p$ are the vectors in V then span $\{ v_1, v_2, v_3, \ldots v_p\}$ is a subspace of V.

(OR)

b) Explain i) Linear transformation ii) Basis and dimension of vector space.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) Solve the linear system using elementary row operations

$$-3x_1 + 2x_2 + 4x_3 = 12$$

$$x_1 - 2x_3 = -4$$

$$2x_1 - 3x_2 + 4x_3 = -3$$ **(OR)**

b) Does the linear homogenous system have any nontrivial solutions?

$$3x_1 + x_2 - 9x_3 = 0$$

$$x_1 + x_2 - 5x_3 = 0$$

$$2x_1 + x_2 - 7x_3 = 0$$

17. a) Show that the every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix. **(OR)**

b) Show that if $\bar{A}$ and $\bar{B}$ be the conjugates of A and B respectively then show that i) $\overline{(kA)} = \bar{k}\bar{A}$, k being any complex numbers. ii) $\overline{(A+B)} = \bar{A} + \bar{B}$.

(CONT…3)

18. a) Find the adjoint of the matrix A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and verify the theorem A(adj A)=(adj A) A= $|A|I_n$.

(OR)

b) Find the inverse of $\begin{bmatrix} A & B \\ C & O \end{bmatrix}$, where B,C are singular..

19. a) Determine the characteristic roots and corresponding characteristic vectors of the given matrix. A= $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

(OR)

b) Prove that Cayley Hamilton theorem.

20. a) If S is any subset of a finite dimensional vector space, then prove that $(S^{\circ})^{\circ}$ is the subspace spanned by S.

(OR)

b) Let V and W be vector spaces over the field F and let t be a linear transformation from V into W. The null space of T^t is the annihilator of the range of T. If V and W are finite dimensional then prove that i) rank (T^t) = rank (T) ii) the range of T^t is the annihilator of the null space of T.

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