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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS313

REG.NO.:

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2024

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: III **TIME: 3 HOURS**

COMBINATORICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. $c(8,5) =$ _______.

   a. 54 b. 56 c. 52 d. 50

2. If $A_1 = \{1,2\}, A_2 = \{4\}, A_3 = \{1,3\},$ and $A_1 = \{2,3,4\}$, then distinct representatives for the sets A_i.

   a. 1, 4, 3, 2 b. 1, 4, 3 c. 1, 4, 2 d. 4, 3, 2

3. A simple tree is defined to be a tree in which each vertex is of degree _________.

   a. < 3 b. > 3 c. ≤ 3 d. ≥ 3

4. If $A = \{1,2,3\}$ and $B = \{2,3,4\}$. Then $|A \cup B| =$ _____.

   a. 3 b. 2 c. 4 d. 6

5. $\binom{21}{2} / \binom{5}{2} =$ ________.

   a. 21 b. 5 c. 2 d. 13

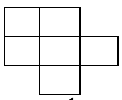
ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. Find $f(10,3)$.

7. A pack of 52 cards is divided among 4 people so that each gets 13 cards(as in bridge). How many such deals are possible?

8. If $a_n = 4(a_{n-1} - a_{n-2})$ for each $n \geq 3$, and if $a_1 = 0, a_2 = 4$, find a_n.

9. Find the rook polynomial for the following board



10. If $n = 3$ and spheres of radius $\frac{1}{2}\sqrt{3}$ are centred at each lattice point, how many spheres will touch any given sphere?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K3)

11. a) Solve the recurrence relation $a_n = na_{n-1}$, subject to the condition $a_1 = 1$.

(OR)

b) 30 girls, including Miss U.K., enter a Miss World competition. The first 6 places are announced. (a) How many different announcements are possible? (b) How many if Miss U.K. is assured of a place in the first six?

12. a) If $r < n$, any $r \times n$ Latin rectangle can be extended to an $(r + 1) \times n$ Latin rectangle.

(OR)

b) State and prove the exchange property of the Gale's optimal assignment problem.

13. a) Find the recurrence relation $u_n, n \geq 2$, two types of tree have to be considered.

(OR)

b)Calculate s_5.

14. a) Find the rook polynomial for an ordinary 4×4 board.

(OR)

b) If A is a square (0, 1)-matrix and if A satisfies (6, 5) with $k > \lambda$, then $AA^{'} = (k - \lambda)I + \lambda J$ also holds.

15. a) Define Lattice along with two examples.

(OR)

b) If E, F are octads and $E \cap F = \phi$, then prove that $(E + F)^{'}$ is also an octad.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Describe the second approach to find a general formula for $f(n, k)$.

(OR)

b) Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.

17. a) If a graph has $2n$ vertices, each of degree $\geq n$, then prove that the graph has a perfect matching.

(OR)

b) State and prove the Konig-Egervary max-min theorem.

18. a) Find the Fibonacci sequence for the equation $x^2 = x + 1$.

(OR)

b) Suppose that, there are 4 colours available. How many colourings of the k golf balls are possible if there must be an odd number of objects coloured with the first colour?

19. a) The manager of a firm has 5 employees to be assigned to 5 different jobs. The men are A, B, C, D, E and the jobs are a, b, c, d, e. He considers that A is unsuited for job b and c, B unsuited for a and c, C unsuited for b, d and e, D suited for all and E unsuited for d. In how many ways can he assign the jobs to men suited to them?

(OR)

b) State and prove the Fisher's result.

20. a) i).Explain a Steiner system,

ii).Prove that the number of *m*-element sets in an *S(l, m, n)* is $\binom{n}{l} / \binom{m}{l}$.

iii).Give application.

(OR)

b) Show that no dodecad contains an octad.

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