(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS312

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: III **TIME : 3 HOURS**

FUNCTIONAL ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. A complete normed linear space is a _______ space.
   a) compact b) banach c) continuous d) hilbert

2. P is said to be an orthogonal projection if ____________
   a) R(P) ⊥ N(P) b) R(P) = N(P) c) R(P) = 0 d) N(P) = 0.

3. Every countable orthonormal basis of a Hilbert Space is a _________ basis.
   a) Parseval b) Schwarz c) Schauder d) orthogonal

4. The sub-linear functional is also known as ____________ functional.
   a) linear b) concave c) convex d) bounded

5. If P is a projection on X and if N(P) and R(P) are closed subspaces of X then P is _________
   a) continuous b) bounded c) converges d) not continuous

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define a symmetric sesquilinear form.
7. What is the condition for absolute convergent series?
8. Define normed linear space.
9. If $x \in X$ is such that $f(x) = 0$ for every $f \in X'$ then what about the value of x?
10. What are the conditions required for a pointwise bounded set to be Uniformly bounded?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State and Prove the Cauchy – Schwarz inequality in K^n.

(OR)

b) Prove that any two norms on a finite dimensional linear space are equivalent.

12. a) Prove that interior of a proper subspace of a normed linear space is empty.

(OR)

b) State and Prove the Projection Theorem.

13.a) Let X and Y be normed linear spaces. If X is finite dimensional, then prove that every linear operator A: X → Y is continuous.

(OR)

b) Let X and Y be normed linear spaces. Then show that the map A → ν_A is a norm on *B*(X, Y).

14. a) Let X and Y be normed linear spaces and A ∈ *B*(X, Y) then prove that ||A'|| = ||A||.

(OR)

b) Let X be a normed linear space and J: X → X^{II} be defined by J(x) = ϕ_x where ϕ_x(f) = f(x) for every f ∈X^{I} and x ∈ X. Then prove that J is a linear isometry.

15. a) Let X be a normed linear space and E subset of X. Then show that E is bounded in X if and only if f (E) is bounded in K for every f ∈ X'.

(OR)

b) Let X and Y be normed linear spaces and A ∈ *B*(X, Y). If A is surjective then show that there exists c > 0 such that for every y ∈ Y there exists x ∈ X such that Ax = y, ||x|| ≤ c||y||.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Derive the Polarization Identities.

(OR)

b) Let X_0 be a closed subspace of a normed linear space X. If X is a Banach Space then prove that X/X_0 is a Banach Space.

17. a) State and Prove Baire Category Theorem.

(OR)

b) Let X be a normed linear space then prove the following are equivalent:

(i) X is finite dimensional

(ii) The unit sphere {x ∈ X: ||x|| = 1} is compact.

(iii) The unit sphere {x ∈ X: ||x|| = 1} is totally bounded.

18. a) Let X and Y be normed linear spaces and A: X→Y be a linear operator then prove that the following are equivalent:

(i) A is continuous at 0, (ii) A is continuous at every x ∈ X

(iii) A is uniformly continuous (iv) there exists c > 0 such that ||Ax|| ≤ c||x|| for all x ∈ X.

(v) {Ax : x ∈ X, ||x|| = 1}is a bounded subset of Y

(vi) For every bounded subset E ⊆ X, the set {Ax : x ∈ E} is bounded in Y.

(OR)

b) (i) State and Prove the Bessel's inequality.

(ii) Let E be an orthonormal set in an inner product space X. Then prove that for every x ∈ X the set E_x = {u ∈ E : < x, u > ≠ 0}is a countable set.

19. a) State and Prove Hahn – Banach Extension Theorem.

(OR)

b) Let X be a normed linear space and **Ω** be a dense subset of X. Then show that X is linearly isometric with the subspace of $\ell^\infty(\Omega)$.

20.a) State and Prove the Uniform Boundedness Principle.

(OR)

b) If X and Y are Banach Spaces, then prove that every closed linear operator A: X→Y is continuous.

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