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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS104

REG.NO. :

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2024

COURSE NAME: M.Sc. MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: I **TIME: 3 HOURS**

ORDINARY DIFFERENTIAL EQUATIONS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If two differentiable functions x_1 and x_2 defined on I are ________ on I then value of Wronskin is $W[x_1(t), x_2(t)] = 0$ on I.

   a) linearly dependent b) linearly independent c) both (a) and (b) d) none of these

2. The value of $J_0'(t) =$ ______ .

   a) $-J_0(t)$ b) $J_0(1)$ c) $-J_1(t)$ d) $J_1(t)$

3. Eigen vectors corresponding to distinct Eigen values are _______.

   a) linearly dependent b) linearly independent c) both (a) and (b) d) none of these

4. Lipschitz condition is defined as _______.

   a) $|f(t,x_1) + f(t,x_2)| \geq K|x_1 + x_2|$ b) $|f(t,x_1) + f(t,x_2)| \leq K|x_1 + x_2|$

   c) $|f(t,x_1) - f(t,x_2)| \geq K|x_1 - x_2|$ d) $|f(t,x_1) - f(t,x_2)| \leq K|x_1 - x_2|$

5. The boundary conditions $x(A) = x(B)$ and $x'(A) = x'(B)$ are known as ________boundary condition.

   a) Singular b) Periodic c) Regular d) Irregular

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define Wronskian W of the function.
7. Write the Rodrigues's formula.
8. Define Characteristic equation.
9. State Hille-Wintner theorem.
10. Define singular linear boundary conditions.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11. a) Determine the Wronskian W of the functions $e^t, \cos t, \sin t$.

(OR)

b) Obtain the general solution of $x'' + x' - 6x = 0$.

12.a) State and prove orthogonal property of Legandre polynomial.

(OR)

b) Prove that $\frac{d}{dt}\left[t^p J_p(t)\right] = t^p J_{p-1}(t)$.

(CONTD….2)

13.a) Determine a fundamental matrix for $x' = Ax$, where $A = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}$,

(OR)

b) Find the solution for a system of equations $x_1' = 5x_1 - 2x_2$, $x_2' = 2x_1 + x_2$

.

14.a) Prove that $x'' + x = 0$ is oscillatory.

(OR)

b) State and prove Gronwall inequality.

15.a) Define (i) linear homogeneous BVP and (ii) non- linear homogeneous BVP.

(OR)

b) State with reason whether the given BVP $x'' + |x| = 0, 0 \le t \le a \prec \pi, x(0) = 0, x(a) = 1$. is linear homogeneous and non-linear homogeneous .

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Let $b_1, b_2 \cdots, b_n ; I \to R$ be continuous function in $L(x)(t) = 0, t \in I$. Let $\phi_1, \phi_2 \cdots, \phi_n$ be n linear independent of $L(x)(t) = 0, t \in I$ exiting on I and $t_0 \in I$ then show that

$$W(t) = \exp\left[-\int_{t_0}^{t} b_1(s)ds\right] W(t_0) : t_0, t \in I .$$

(OR)

b) Solve the IVP $x'' + x' + 2x = 0$, $x(0) = 0, x'(0) = 2$.

17.a) If P_n are Legendre polynomials, then show that $\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}$.

(OR)

b) Find the general solution of $tx'' + (1-t)x' + nx = 0, t > 0$.

18. a) Determine e^{tA} and a fundamental matrix for the system $x' = Ax$ where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(OR)

b) Let $\phi(t)$ be a fundamental matrix for the system $x' = A(t)x(t), t \in I$ then shown that

$\psi(t) = \phi(t)\int_{t_0}^{t} \phi(s)b(s)ds$, $t \in I$ is a solution of initial value problem $x' = A(t)x + b(t)$, $x(t_0) = 0$.

19.a) State and Prove Picard's Theorem.

(OR)

b) State and Prove Strurm's comparison theorem.

20.a) Solve the BVP $x'' + \lambda x = 0, x(0) = 0, x'(1) = 0$

(OR)

b) show that *G(t,s)* be given by the relation $\begin{cases} -y(t)\,z(s)/A & if\ t \le s, \\ -y(s)\,z(t)/A & if\ t \ge s. \end{cases}$ Then x(t) is a solution of $L(x) + f(t) = 0,\ a \le t \le b$ if and only if $x(t) = \int_a^b G(t,s)\, f(s)ds$.

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