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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS511

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: V **TIME : 3 HOURS**

PART - III
REAL ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. ______ is rational.
   (a) e (b) π (c) $\sqrt{n}$ (d) $\frac{1}{2}$
2. Two functions F and G are equal iff______.
   (a) F and G have the same domain
   (b) $F(x) = G(x)$ *for every x in domain of F*
   (c) F and G have the different domain
   (d) both a & **b**
3. If A is open and B is closed then _______
   (a) $A - B$ is closed (b) $B - A$ is open (c) $A - B$ is open (d) $B - A$ is not defined
4. _________ is called discrete metric space.
   (a) $d(x, y) = |x - y|$ (b) $d(x, y) = 0 \text{ if } x = y$ (c) $d(x, y) = 1 \text{ if } x \neq y$ (d) b & c
5. ____ is not complete.
   (a) every Euclidean space R^k (b) R' (c) the subspace $T = (0,1]$ of R' (d) R^n

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Write approximation property.
7. Define composite functions.
8. Definition of closed set.
9. What is called accumulation point.
10. What is called decreasing sequence .

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State and prove the additive property of the supremum.

(OR)

b) State and prove comparison property.

12. a) State and prove triangular inequality.

(OR)

b) Prove that every subset of a countable set is countable.

13. a) Prove that the set S of intervals with rational end points is a countable set.

(OR)

b) Prove that the set Q of all rational numbers is a countable set .

(CONTD……2)

14. a) Prove that a set S in R^n is closed iff it contains all its adherent points.

(OR)

b) State and prove Lindelof covering theorem.

15. a) Prove that in any metric space (S, d) assume $x_n \to p$ and let $T = \{x_1, x_2, ...\ ....\}$ be the range of $\{x_n\}$. Then prove that

(i) T is bounded

(ii) P is an adherent point of T.

(OR)

b) Prove that in any metric space (S, d), every compact sub set T is complete.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) (i) Assume $x \geq 0$ then for every integer $n \geq 1$ there is a finite decimal $r_n = a_0 a_1 a_2 ....\ a_n$ such that $r_n \leq x < r_n + \frac{1}{10^n}$

(OR)

b) State and prove unique factorization theorem.

17. a) State and prove Cauchy-Schwartz inequality.

(OR)

b) Prove that the set of all real numbers is uncountable.

18. a) Prove that if F is a countable collection of countable sets, then the union of all sets in F is also a countable set.

(OR)

b) State and prove representation theorem for open sets on the real line

19.a) Let f and g be complex-valued functions defined on a sub set A of a metric space (S, d). Let p be an accumulation point of A and assume that $\lim_{x\to p} f(x) = a$, $\lim_{x\to p} g(x) = b$

Then prove that $(i)\ \lim_{x\to p}[f(x) \pm g(x)] = a \pm b$

$(ii)\ \lim_{x\to p}[f(x)\, g(x)] = a.b$

$(iii)\ \lim_{x\to p}[f(x)/g(x)] = a/b \quad if \quad b \neq 0$

(OR)

b) State and prove Heine-Borel theorem.

20. a) Let f and g be complex-valued functions defined on a sub set A of a metric space (S, d). Let p be an accumulation point of A and assume that $\lim_{x\to p} f(x) = a$, $\lim_{x\to p} g(x) = b$

Then prove that $(i)\ \lim_{x\to p}[f(x) \pm g(x)] = a \pm b$

$(ii)\ \lim_{x\to p}[f(x)\, g(x)] = a.b$

$(iii)\ \lim_{x\to p}[f(x)/g(x)] = a/b \quad if \quad b \neq 0$

(OR)

b) In Euclidean space R^k prove that every Cauchy sequence is convergent.

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