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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS206

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : MAY-2024

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: II **TIME : 3 HOURS**

MATHEMATICAL STATISTICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. Two socks are selected at random and removed in succession from a drawer containing five brown socks and three green socks. List the elements of the sample space.
   a) BB,BG,GB b) BG,GB,GG c) BB,BG,GB,GG d) BB,GG
2. What is the probability that $\bar{X}$ will take on a value between $\mu - c$ and $\mu + c$?
   a) atleast $1 - \frac{\sigma^2}{nc^2}$ b) atmost $1 - \frac{\sigma^2}{nc^2}$ c) = $1 - \frac{\sigma^2}{nc^2}$ d) = $1 + \frac{\sigma^2}{nc^2}$
3. If X has the binomial distribution with parameters n and θ, then the sample proposition, $\frac{X}{n}$, is which kind of estimator of θ?
   a) biased b) unbiased c) equal d) unequal
4. If a random sample of size n=20 from a normal population with variance 225 has the mean 64.3, then what is the 95% confidence interval for the population mean?
   a) (56.7, 69.8) b) (57.6, 68.9) c) (50.7, 77.9) d) (57.7,70.9)
5. Suppose that the manufacturer of a new medication wants to test the null hypothesis $\theta = 0.9$ against the alternative hypothesis $\theta = 0.6$. His test statistic is X, the number of successes in 20 trials and he will accept the null hypothesis if $x > 14$; otherwise, he will reject it. What is α?
   a) 0.0114 b) 0.1255 c) 0.01255 d) 0.114

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Find the distribution function of the total number of heads obtained in four tosses of a Balanced coin.
7. If X has the standard normal distribution, then explain about the distribution of X^2.
8. Define Cramer-Rao inequality.
9. In a random sample, 136 of 400 persons given a flu vaccine experienced some discomfort. Construct a 95% confidence interval for the true proposition of persons who will experience some discomfort from the vaccine.
10. Define type II error.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Two caplets are selected at random from a bottle containing three aspirin, two sedative and four laxative caplets. If X and Y are, respectively, the numbers of aspirin and sedative caplets included among the two caplets drawn from the bottle, compute the probabilities associated with all possible pairs of values of X and Y.

(OR)

(CONTD…….2)

b) Consider the trivariate probability density.

$$f(x_1,x_2,x_3) = \begin{cases} (x_1+x_2)e^{-x_3} & for\ 0 < x_1,x_2 < 1, x_3 > 0 \\ 0 & otherwise \end{cases}.$$

Examine the joint marginal density of X_1 and X_3 and the marginal density of X_1 alone.

12.a) If X_r and X_s are the r^{th} and s^{th} random variables of a random sample of size n drawn from the finite population $\{c_1, c_2, \dots, c_N\}$, then show that $\text{cov}(X_r, X_s) = -\frac{\sigma^2}{N-1}$.

(OR)

b) If $\bar{X}$ and S^2 are the mean and the variance of a random sample of size n from a normal population with mean μ and the variance σ^2, then prove that $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ has the t distribution with n-1 degrees of freedom.

13. a) If S^2 is the variance of a random sample from an infinite population with finite variance, then examine $E(S^2)$.

(OR)

b) Show that $Y = \frac{1}{6}(X_1 + 2X_2 + 3X_3)$ is not a sufficient estimator of the Bernoulli parameter θ.

14.a) A study has been made to compare the nicotine contents of two brands of cigarettes. Ten cigarettes of Brand A had an average nicotine content of 3.1 milligrams with a standard deviation of 0.5 milligrams, while eight cigarettes of Brand B had an average nicotine content of 2.7 milligrams with a standard deviation of 0.7 milligrams. Assuming that the two sets of data are independent random samples from normal populations with equal variances. Find 98% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$.

(OR)

b) In 16 test runs the gasoline consumption of an experimental engine had a standard deviation of 2.2 gallons. Examine a 99% confidence interval for variance which measures the true variability of the gasoline consumption of the engine.

15.a) Suppose that it is known from experience that standard deviation of the weight of 8-ounce packages of cookies made by a certain bakery is 0.16 ounce. To check whether its production is under control on a given day, that is, to check whether the true average weight of the packages is 8 ounces, employees select a random sample of 25 packages and find their mean weight is $\bar{x} = 8.091$ ounces. Since the bakery stands to lose money when $\mu > 8$ and the customer loses out when $\mu < 8$, test the null hypothesis $\mu = 8$ against the alternative hypothesis $\mu \neq 8$ at the 0.01 level of significance.

(OR)

b) If x =4 of n =20 patients suffered serious side effects from a new medication, compare the null hypothesis $\theta = 0.5$ against the alternative hypothesis $\theta \neq 0.5$ at 0.05 level of significance.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} x+y & for\ 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

, then examine the joint distribution function of these two random variables.

(OR)

(CONTD …. 3)

b) Given the joint probability density $f(x,y) = \begin{cases} 4xy & for\ 0 < x < 1, 0 < y < 1 \\ 0 & elsewhere \end{cases}$, then determine the marginal densities of X and Y and the conditional density of X given $Y = y$.

17.a) State and prove central limit theorem.

(OR)

b) Investigate the probability density function of F distribution.

18.a) Show that $\bar{X}$ is a minimum variance unbiased estimator of the mean of a normal population.

(OR)

b) If $X_1, X_2, \ldots, X_n$ constitute a random sample of size n from a normal population with mean μ and variance σ^2, determine the joint maximum likelihood estimates of these two parameters.

19.a) If 132 of 200 male voters and 90 of 159 female voters favor a certain candidate running a governor of Illinois, examine the 99% confidence interval for the difference between the actual proportions of male and female voters who favor the candidate.

(OR)

b) A study has been made to compare the nicotine contents of two brands of cigarettes. Ten cigarettes of Brand A had an average nicotine content of 3.1 milligrams with a standard deviation of 0.5 milligrams, while eight cigarettes of Brand B had an average nicotine content of 2.7 milligrams with a standard deviation of 0.7 milligrams. Assuming that the two sets of data are independent random samples from normal populations with equal variances. Determine a 95% confidence interval for the difference between the mean of the two brands of cigarettes.

20.a) Suppose that 100 tires made by a certain manufacturer lasted on the average 21,819 miles with a standard deviation of 1,295 miles, examine the null hypothesis $\mu = 22{,}000$ miles against the alternative hypothesis $\mu < 22{,}000$ miles at the 0.05 level of significance.

(OR)

b) In the comparison of two kinds of paint, a customer testing service finds that four 1-gallon cans of one brand cover on the average 546 square feet with a standard deviation of 31 square feet, whereas four 1-gallon cans of another brand cover on the average of 492 square feet with a standard deviation of 26 square feet. Assuming that the two populations sampled are normal and have equal variances. Test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative hypothesis $\mu_1 - \mu_2 > 0$ at the 0.05 level of significance.

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ETHICAL PAPER