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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS4E1

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2024

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: IV **TIME : 3 HOURS**

MATHEMATICAL METHODS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. A complex-valued function $K(s,t)$ is called symmetric if _________.
   a. $K(s,t) = K^*(s,t)$ b. $K(t,s) = K^*(s,t)$ c. $K(s,t) = K^*(t,s)$ d. $K(s,t) = K(t,s)$

2. An integral equation is called singular if either the range of integrations is infinite or ________ within the range of integration.
   a. the kernel has singularities b. the kernel has Volterrra
   c. the kernel initial value d. the kernel has bounded values

3. A necessary and sufficient condition for the extremum of the function $\varphi'(0)$ for $\alpha = 0$ isits derivative _______.
   a. $\varphi'(0) \neq 0$ b. $\varphi'(0) = 0$ c. $\varphi''(0) \neq 0$ d. $\varphi''(0) = 0$

4. If a proper field or a central field is formed by family of extremals of a certain variational problem, then it is called ______________.
   a central field b. a proper field c. a strong field d. an extremal field

5. If the _______ method is used to determine the absolute minimum of the functional, then the approximate value of the minimum of the functional is obtained in excess on the curves of the form $y_n = \sum_{i=1}^{n} \alpha_i W_i(x)$.
   a. Ritz method b. direct method
   c. Euler's finite-difference method d. Kantorovich's method

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. When will we say two functions are orthogonal?
7. Reduce the initial value problems $y'' + \lambda y(s) = F(s)$, $y(0) = 1$, $y'(0) = 0$, to a Volterra equation.
8. On what curves can the functional $v[y(x)] = \int_0^{\pi/2} [(y')^2 - y^2]dx$; $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$be extremed?
9. How is Jacobi condition sufficient fora point on the curve?
10. Write the Ostrogradsky equation for the functional.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Solve the Fredholm integral equation of the second kind $g(s) = s + \lambda \int_0^1 (st^2 + s^2 t)\ g(t)dt$.

(OR)

b) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 e^{s-t}\ g(t)dt$.

(CONTD….2)

12.a) Reduce the boundary value problem $y''(s)+\lambda P(s)y=Q(s); \quad y(a)=0, \quad y(b)=0$ to a Fredholm integral equation.

(OR)

b) Solve the integral equation $f(s)=\int_0^{\pi}\frac{g(t)dt}{(\cos t-\cos s)^{1/2}}, \quad 0\le a<s<b\le\pi.$

13.a) State and prove the fundamental lemma of the calculus of variations.

(OR)

b) Derive the differential equation of free vibration of a sting.

14. a) Is the Jacobi condition fulfilled for extremal of the functional $v[y(x)]=\int_0^a (y'^2+y^2+x^2)dx,$ that passes through the points A(0, 0) and B(a, 0)?

(OR)

b) Test for an extremal the functional $v=\int_0^a y'^3 dx, \quad y(0)=0, y(b)=b, \; a>0, b>0.$

15.a) In problems associated with the torsion of a cylinder or prism, investigate the functiona $v[z(x,y)=\iint_D\left[\left(\frac{\partial z}{\partial x}-y\right)^2+\left(\frac{\partial z}{\partial y}+x\right)^2\right]dxdy$ for an extremum.

(OR)

b) Investigate for an extremum the functional $v[z(x,y)=\int_{-a}^{a}\int_{-b}^{b}\left[\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2-2z\right]dxdy$ on the boundary of the integration domain *z* = 0.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Solve the integral equation $g(s)=f(s)+\lambda\int_0^1 (s+t)\,g(t)dt$ and find the eigenvalues.

(OR)

b) Show that the integral $g(s)=f(s)+\frac{1}{\pi}\int_0^{z\pi}[\sin(s+t)]g(t)dt$ possesses no solution for *f(s)= s*, but that it possesses infinitely many solutions when *f(s) = 1*.

17.a) Reduce the boundary value problem $y''+\lambda y=0; \quad y(0)=0, \quad y'(1)+v_2 y(1)=1,$ to a Fredholm integral equation.

(OR)

b) Solve the integral equations (i) $f(s)=\int_a^s \frac{g(t)}{(s^2-t^2)^{\alpha}}dt, \quad 0<\alpha<1; a<s<b,$

(ii) $f(s)=\int_s^b \frac{g(t)}{(t^2-s^2)^{\alpha}}dt, \quad 0<\alpha<1; a<s<b.$

(CONTD…..3)

18. a) Explain the brachistochrone problem. Also find the curve connecting given A and B which is traversed by a particle sliding from A to B in the shortest time.

(OR)

b) Determine the extremal of the functional $v[y(x)] = \int_{-l}^{l} \left(\frac{1}{2} \mu y''^2 + \rho y \right) dx,$ that satisfies the boundary conditions $y(-l) = 0, \quad y'(-l) = 0, \quad y(l) = 0, \quad y'(l) = 0.$

19.a) Test for an extremum for the functional $\int_0^a (6y'^2 - y'^4 + yy')\, dx; \quad y(0) = 0, \quad y(a) = b;$ $a > 0$ and $b > 0.$

(OR)

b) Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = [\varphi_1(x) + \varphi_2(y)](dx^2 + dy^2).$

20.a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x, y)$ inside the rectangle *D*, $0 \le x \le a, \quad 0 \le y \le b,$ that vanishes on the boundary of *D*.

(OR)

b) Find a continuous solution of the equation $\Delta z = -1$ in the domain *D*, which is an isosceles triangle bounded by the straight lines $y = \pm \frac{\sqrt{3}}{3} x$ and $x = b$, which solution vanishes on the boundary of the domain.

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