(FOR THE CANDIDATES ADMITTED (NO. OF PAGES: 2)

DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS613

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2024

COURSE NAME:BSc.- MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: VI **TIME : 3 HOURS**

PART - III

LINEAR ALGEBRA

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. Singular matrix are____________.
   (a) non-invertible (b) invertible
   (c) Both non-invertible and invertible (d)None Of the above

2. Which one of the following is correct?
   (a) R is a vector space over N (b) R is a vector space over C
   (c) R is a vector space over Z (d) R is a vector space over Q

3. If A is $\begin{bmatrix} 8 & 5 \\ 7 & 6 \end{bmatrix}$ then the value of $|A^{11} - A^{10}|$ ______________.
   (a) 0 (b) 1 (c) 120 (d) 121

4. For each linear transformation : $R^2 \to R^2$. Find the matrix representing t relative to the standard basis of R^2 if t is rotation in R^2 counter clock wise by 45^0
   (a) $\begin{bmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (c) $\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$ (d) none of these .

5. Let V be a finite-dimensional vector space over the field F. Each basis for V* is_________.
   (a) The dual of some basis for V. (b) maximal proper subspace of V
   (c) The dual of all basis for V. (d) none of these

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define a Field.
7. Define Vector Space.
8. Define a linear transformation from V into W.
9. Define annihilator.
10. Define transpose of A and give an example.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K3

11. a) If A, B, C are matrices over the field F such that the products BC and A(BC) are defined, then so are the products AB, (AB)C and A(BC) = (AB)C.

(OR)

(b) If A and B are row-equivalent mXn matrices, the homogeneous systems of linear equations AX = 0 and BX = 0 have exactly the same solutions.

(CONTD…..2)

12.(a) Check whether $(1,-2,5)$ is a linear combination of $(1,1,1)$, $(1,2,3)$ and $(2,-1,1)$

(OR)

(b) Determine whether the following set of vectors are Linearly independent or linearly dependent in $V_3(R)$**:** $(1,4,-2), (-2,1,3), (-4,11,5)$

13.a) Let V and W be vector spaces over the field F and let T be a linear transformation from $V\ into\ W$. If T is invertible, then the inverse function T^{-1} is a linear transformation from $W\ onto\ V$.

(OR)

b) Let V be a vector space over the field F ; let U, T_1 and T_2 be linear operators on V ; let c be an element of F. Prove that (i) $= UI = U$;

(ii) $U\,(T_1 + T_2)\; UT_1 + UT_2$; $(T_1 + T_2)U = T_1U + T_2$

14.a) Let $T: V_3 \to V_3$ given by $T(a,b,c) = (3a + c, -2a + b, a + 2b + 4c)$ .Find the matrix representation with respect to the standard basis.

(OR)

b) Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Then prove that $dimW + dimW^0 = dimV$.

15.a) Let V be a finite-dimensional vector space over the field F. For each vector a in V define $L_\alpha(f) = f(\alpha)$, f in V^*. Prove that the mapping $\alpha \to L_\alpha$ is then an isomorphism of Vonto V^{**} .

(OR)

b) If S is any subset of a finite-dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S.

SECTION - C **(4 X 10 = 40 MARKS)**

ANSWER ANY FOUR OUT OF SIX QUESTIONS.

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS FROM Qn.No : 17 to 21)

16. Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as Followsu $+\, v = uv$ for all $u, v \in R^+$; $\alpha u = u^\alpha$For all $u \in R^+$and $\alpha \in R^+$. Determine whether or not R^+ is real vector space.

17. Suppose F is the field of rational numbers, $A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 4 & 0 & -1 \\ 2 & 6 & -1 & 5 \end{bmatrix}$ and perform a finite sequence of elementary row operations on A .

18. If W_1 and W_2 are finite-dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite-dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

19. State and prove Dimension theorem for linear transformation.

20. Find the matrix representation $T: V_2 \to V_2$ given by $T(a,b) = (-b,a)$ with respect to the Basis (1,2), (1,-1).

21. Let V and W be finite-dimensional vector spaces over the field F. Let $\mathfrak{B}$ be an ordered basis for V with dual basis $\mathfrak{B}^*$, and let $\mathfrak{B}'$ be an ordered basis for W with dual basis $\mathfrak{B}'^*$. Let T be a linear transformation from V into W ; let A be the matrix of T relative to $\mathfrak{B}$, $\mathfrak{B}'$ and let B be the matrix of T^t relative to $\mathfrak{B}'^*, \mathfrak{B}^*$. Then prove that $B_{ij} = A_{ji}$.

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ETHICAL PAPER