

(FOR THE CANDIDATES ADMITTED

21UMS6E1

DURING THE ACADEMIC YEAR 2021 ONLY)

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS :MAY-2024

COURSE NAME : B.Sc.-MATHEMATICS

MAXIMUM MARKS: 70

SEMESTER: VI

TIME : 3 HOURS

## PART - III

## DISCRETE MATHEMATICS

## SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS.

(K1)

- The value of  $F_4$  of the Fibonacci numbers using recursion is \_\_\_\_\_.  
(a) 5 (b) 4 (c) 3 (d) 1
- The value of  $p \vee (p \wedge q)$  is \_\_\_\_\_.  
(a)  $p$  (b)  $q$  (c)  $p \wedge q$  (d)  $p \vee q$
- If for every  $a, b \in X$ , both  $a \vee b$  and  $a \wedge b$  exists in a Poset  $(X, \leq)$  then the Poset is \_\_\_\_\_.  
(a) Boolean Algebra (b) Lattice (c) Modular (d) Equivalence relation
- A graph in which every vertex has same degree is \_\_\_\_\_.  
(a) Irregular (b) regular (c) multi graph (d) connected graph
- If  $\Sigma = \{0,1,2\}$  and the string  $w = 01212$  then the length of  $w$  is \_\_\_\_\_.  
(a) 2 (b) 3 (c) 5 (d) 0

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

- Define characteristics equation of the homogeneous equation of order  $n$ .
- Define elementary product.
- If  $D(n)$  denotes the Lattice of all positive divisors of the integer  $n$ , draw the Hasse diagram of  $D(10)$ .
- Define intersection of two graphs  $G_1$  and  $G_2$ .
- Define type  $K$ - language.

## SECTION – B

(5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

- Find the recurrence relation satisfying  $y_n = A(3)^n + B(-4)^n$ .  
(OR)
  - Find  $f(n)$  if  $f(n) = 7f(n-1) - 10f(n-2)$  given that  $f(0) = 4$  and  $f(1) = 17$ .
- Show that  $(P \rightarrow Q) \Rightarrow (\neg Q \rightarrow \neg P)$ .  
(OR)
  - Obtain a disjunctive normal form of  $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$ .
- Let  $(L, \leq)$  be a lattice. For any  $a, b \in L$ . If  $a \leq b$  then prove that  $a \vee b = b$ .  
(OR)
  - In a Boolean algebra  $L$ , prove that  $(a \wedge b)' = a' \vee b'$  for all  $a, b \in L$ .

(CONTD.....2)

14.a) Prove that the number of vertices of odd degree in a graph is always even.

(OR)

b) Draw the complete graph  $K_6$  and also find the number of edges in the graph  $K_{15}$ .

15.a) Let  $\Sigma = \{0,1\}$ . Show that the following expressions are regular expressions over  $\Sigma = \{0,1\}$

(a)  $0^*(0-1)$  (b)  $00^*(0+1)^*$  (c)  $(01)^*(01+1)^*$ .

(OR)

b) Let  $V = \{S, A, B\}, \Sigma = \{a, b\}$  starting symbol  $S$  and production

$P = \{S \rightarrow aABa, A \rightarrow baABb, B \rightarrow Aab, aA \rightarrow baa, bBb \rightarrow abab\}$ .

Find the language generated by  $G = (V, \{a, b\}, S, P)$ .

### SECTION - C

(4 X 10 = 40 MARKS)

**ANSWER ANY FOUR OUT OF SIX QUESTIONS**

**(16<sup>th</sup> QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS**

**(FROM Qn. No : 17 to 21)**

**(K4 (Or) K5)**

16. Let  $L$  be a complemented, distributive lattice. For  $a, b \in L$ ,

Prove that the following are equivalent.

- (i)  $a \leq b$
- (ii)  $a \wedge b' = 0$
- (iii)  $a' \wedge b = 1$
- (iv)  $b' \leq a'$

17. Using the generating function solve the difference equation  $y_{n+2} - y_{n+1} - 6y_n = 0$  given  $y_1 = 1, y_0 = 2$ .

18. Show that if  $p \rightarrow q, q \rightarrow r, \neg(p \wedge r)$  and  $p \vee r$ , then  $r$ .

19. In a distributive lattice, prove that the following are equivalent

- (i)  $a \wedge b \leq x \leq a \vee b$
- (ii)  $x = (a \wedge x) \vee (b \wedge x) \vee (a \wedge b)$ .

20. Prove that a simple graph  $G$  with  $n$  vertices and  $k$  components cannot have more than  $\frac{1}{2}(n-k)(n-k+1)$  edges.

21. Design a finite – state machine that performs serial addition.

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