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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS314

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: III **TIME : 3 HOURS**

GRAPH THEORY

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **K1**

1. A trivial graph has ___ vertex.

   a) 1 b) 2 c) finite d) no

2. A connected graph is a block if has _____ cut vertices.

   a) 1 b) 2 c) finite d) no

3. Every augmenting path is ______.

   a) covering b) perfect matching c) alternating path d) none of the above

4. Every critical graph is a _____.

   a) block b) complete graph c) 4-chromatic d) polinomial

5. If G is a plane graph then $\sum d(f)$__________.

   a) ε b) 2ε c) 3ε d) 5ε

ANSWER THE FOLLOWING QUESTIONS IN ONE OR TWO SENTENCES.

6. Show that in any graph the number of vertices of odd degree is always even.
7. Define Hamiltonian graph.
8. Define an M augmenting path
9. Draw a proper 3 edge colourable graph.
10. Define indegree of a graph.

SECTION – B **(5 X 3 = 15 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

11. a) If G is a tree, then show that $\varepsilon = \nu - 1$.

(OR)

b). Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

12. a) If G is a block with $v \geq 3$, then prove that any two edges of G lie on a common cycle.

(OR)

b) Prove that the closure of a graph C(G) is well defined.

(CONTD…..2)

13. a) Prove that a matching M in G is a maximum matching iff G contains no M augmenting path.

(OR)

b) Verify that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

14. a) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

(OR)

b) Prove that if G is K critical, then $\delta \geq K - 1$

15. a) Prove that K5 is nonplanar

(OR)

b) If G is a connected plane graph, then prove that $\upsilon - \varepsilon + \phi = 2$.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) Prove that a graph is bipartite if and only if it contains no odd cycle.

(OR)

b) Prove that $\tau(K_n) = n^{n-2}$

17. a) Prove that any graph G with $\upsilon \geq 3$ is 2-connected iff any two vertices are connected by at least two internally disjoint paths.

(OR)

b) Show that a nonempty connected graph is eulerian iff it has no vertices of odd degree.

18. a) Prove that G has a perfect matching iff $o(G - S) \leq |S|$ for all $S \subset V$.

(OR)

b) Prove that $\alpha' + \beta' = \upsilon$, for $\delta > 0$

19.a) Prove that if G is simple then $\chi' = \Delta \ or \ \chi' = \Delta + 1$

(OR)

b) Prove that if G is a connected simple graph and is neither an odd cycle nor a complete graph, then $\chi \leq \Delta$.

20. a) Prove that a graph is planar iff it contains no subdivision of K_5 or $K_{3,3}$

(OR)

b) Prove that a digraph D contains a directed path of length $\chi - 1$.

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