(FOR THE CANDIDATES ADMITTED 23UAI1A1

DURING THE ACADEMIC YEAR 2023 ONLY) REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

BSC COMP.SCIENCE WITH AI &ML **MAXIMUM MARKS: 75**

SEMESTER:I **TIME : 3 HOURS**

PART - III

23UAI1A1– : INTRODUCTION TO LINEAR ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

1. The essential information of a linear system can be recorded compactly in a rectangular array called a________

a) Matrix b) Polynomial c) Array d) equation

2. A _____________matrix is a square n x n matrix whose nondiagonal entries are zero.
a) diagonal b) unit c)zero d)identity

3. The set of all linear combinations of the row vectors is called the_________
a) row space b)Column space c) vector space d) null space

4. An _______of an n x n matrix A is a nonzero vector **x** such that A**x** = λ**x** for some scalar λ.
a) eigenvector b) eigenvalue c) eigenspace d) subspace

5.The inner product is also referred as ______

a) dot product b) length c) vector d) norm

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)
(Qn. No. 6 - 10)

6. Mention the properties of rectangular matrix in echelon form.

7. Recall block diagonal matrix.

8. Define Vector space.

9. Determine if the following matrix is diagonalizable.

$$A = \begin{pmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{pmatrix}$$

10 What is orthogonal set?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Determine if the following system is consistent:

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$4x_1 - 8x_2 + 12x_3 = 1$$

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(CONT…2)

(OR)

b) Given $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ find $4\mathbf{u}$, $(-3)\mathbf{v}$, and $4\mathbf{u} + (-3)\mathbf{v}$.

12. a) Compute AB, where $A = \begin{pmatrix} 2 & 3 \\ 1 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{pmatrix}$

(OR)

b) Let $A = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$.

Verify that $AB = \text{col}_1(A)\,\text{row}_1(B) + \text{col}_2(A)\,\text{row}_2(B) + \text{col}_3(A)\,\text{row}_3(B)$

13. a) Given **v**1 and **v**2 in a vector space V , let H = Span {**v**1,**v**2}.
Show that H is a subspace of V .

(OR)

b) i) If A is a 7 x 9 matrix with a two-dimensional null space, what is the rank of A?
ii) Could a 6 x 9 matrix have a two-dimensional null space?

14. a) Let $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ Are **u** and **v** eigenvectors of A?

(OR)

b) Suppose $\mathcal{B} = \{b_1, b_2\}$ is a basis for V and $\mathcal{C} = \{c_1, c_2, c_3\}$ is a basis for W . Let T : V → W be a linear transformation with the property that $T(b_1) = 3c_1 - 2c_2 + 5c_3$ and $T(b_2) = 4c_1 + 7c_2 - c_3$ Find the matrix M for T relative to $\mathcal{B}$ and $\mathcal{C}$.

15. a) Compute **u.v** and **v.u** for $\mathbf{u} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$

(OR)

b) The distance from a point **y** in R" to a subspace W is defined as the distance from **y** to the nearest point in W . Find the distance from **y** toW = Span {$\mathbf{u}_1$, $\mathbf{u}_2$}, where

$\mathbf{y} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$ $\mathbf{u}_1 = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$ $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K4 (Or) K5)

16. a) Compute A**x**, where $A = \begin{pmatrix} 2 & 3 & 4 \\ -1 & 5 & -3 \\ 6 & -2 & 8 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

(OR)

b) Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set

$3x1 + 5x2 - 4x3 = 0$
$-3x1 - 2x2 + 4x3 = 0$
$6x1 + x2 - 8x3 = 0$

17. a) Find the inverse of the matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$, if it exists.

(OR)

b) Find an LU factorization of

$$A = \begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{pmatrix}$$

18. a) Find a spanning set for the null space of the matrix

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$

(OR)

b) Find bases for the row space, the column space, and the null space of the matrix

$$A = \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}$$

19. a) Let $A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$ An eigenvalue of A is 2. Find a basis for the corresponding eigenspace

(OR)

b) Let $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$ Find a formula for A^k, given that $A = PDP^{-1}$,

where $P = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$

20.a) Let $\mathbf{u}_1 = \begin{pmatrix} 2 \\ \mathbf{5} \\ -1 \end{pmatrix}$ $\mathbf{u}_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Observe that { $\mathbf{u}_1$, $\mathbf{u}_2$} is an orthogonal basis for W D Span {$\mathbf{u}_1$, $\mathbf{u}_2$}. Write **y** as the sum of a vector in W and a vector orthogonal to W

(OR)

b) Let $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ Then {$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$} is clearly linearly independent and thus is a basis for a subspace W of R^4. Construct an orthogonal basis for W

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