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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2020 ONLY)

20PMS417

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : JULY-2022

M. Sc.-MATHEMATICS — MAXIMUM MARKS: 70

SEMESTER : IV — TIME : 3 HOURS

CONTROL THEORY

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.

1. The system $\dot{x}$= A(t)x, y(t) = H(t)x(t) is _____on an interval t ∈ $[0,T]$ if y(t) = H(t)x(t) = 0 implies x (t) = 0, t ∈ $[0,T]$.
   a) linearly independent b) linearly dependent c) observable d) singular
2. The system described by $\dot{x_1}= -x_1 + u_1, \dot{x_2} - 2x_2 + u_1 + u_2$ is __________.
   a) observable b) controllable c) nonsingular d) singular
3. The system $\dot{x}$ = Ax is asymptotically stable if all the eigenvalues of A have __________.
   a) positive real parts b) negative real parts c) positive values d) negative values
4. The system $\dot{x}$ = (A + BK) x is called a/an______________.
   a) an open loop system b) a closed loop system c) positive definite d) stable
5. The optimal control must ___________ the Hamiltonian.
   a) minimize b) maximize
   c) both maximize and minimize d) neither maximize nor minimize

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. K2

6. Define a reconstruction Kernel.
7. Define completely controllable system.
8. Define uniformly stable solution
9. Define stabilizability.
10. Write down the linear time invariant system.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

11. a) Prove that there exists a reconstruction kernel R(t) on [0, T] if and only if $\dot{x}$ = A(t)x, y(t) = H(t)x(t) is observable on [0, T]

(OR)

b) Check the observability of the following system $t^2\ddot{x} + t\,\dot{x} - x = 0$ with observation y = $\dot{x}$

12.a) Prove that the system $\dot{x}$ = A(t)x+B(t)u is controllable on [0, T] if for each vector x∈$\Re^n$ there is a control u∈L_m^2 [0, T] which steers 0 to x, during[0, T]

(OR)

b) Show that the following system is completely controllable on [0, T]:
$\dot{x}_1$= -x_1 +x_2 + (cost)u_1 + (sint)u_2 +$\frac{10x_1}{1+x_1^2+x_2^2+u_1^2}$

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13. a) Prove that the system $\dot{x} = Ax$ is asymptotically stable iff every eigen value of A has negative real part.

(OR)

b) Show that the following system is stable: $\dot{x} = Ax,$ where A = $\begin{bmatrix} -1 & 0 & 0 \\ -2 & -1 & 2 \\ -3 & -2 & -1 \end{bmatrix}$

14.a) Prove that if the system $\dot{x} = Ax + Bu,$ x $\in R^m$, u $\in R^n$ is controllable, then it is stabilizable.

(OR)

b) Suppose there are m × n matrices K_1, K_2 such that $(A + BK_1)$ and $-(A + BK_2)$ are stability matrices. Prove that the system $\dot{x} = Ax + Bu,$ x $\in R^m$, u $\in R^n$ is controllable.

15. a) Prove that if K(t) is the solution of the Riccati equation $\dot{K}(t) + K(t)A(t) + A^*(t)K(t) - K(t)S(t)K(t) + Q(t) = 0$ and if K(t) = F, then K(t) is symmetric for all t$\in$[0, T] .

(OR)

b) Find the optimal control u for the system$\dot{x}(t) = x(t) + u, \quad x(0) = x_0$ and the cost functional $J = \int_0^1 (3x^2(t) + u^2(t)]dt.$

SECTION - C **(4 X 10 = 40 MARKS)**

ANSWER ANY FOUR OUT OF SIX QUESTIONS.

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) **(K4 (Or) K5)**

16. Prove that the constant coefficient system $\dot{x}$ = Ax, y = Hx is observable on an arbitrary interval [0, T] if and only if for some k , 0< k ≤ n , the

$$\textbf{rank} \begin{bmatrix} H \\ HA \\ \cdot \\ \cdot \\ \cdot \\ HA^{k-1} \end{bmatrix} \textbf{= n}$$

17. Prove that the observed linear system $\dot{x}$ = A(t)x, y(t) = H(t)x(t) is observable on [0, T] if and only if the observability Grammian matrix is positive definite

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18. Prove that the constant coefficient control system $\dot{x}$ = Ax + Bu is controllable if and only if rank$[B, AB,\ldots,A^{n-1}B]$ = n.

19. Let X(t) be a fundamental matrix of the system $\dot{x}$ = A(t)x(t), where A(t) is a n × n matrix on J.
Prove the following: The system is stable iff there exists a constant k > 0 with $\|x(t)\| \le K, t \in J$.
(ii) The system is uniformly stable iff there exists a constant k > 0 with $\|x(t,s)\| \le K$,**0 ≤ s ≤** t<∞
(iii) the above system is uniformly asymptotically stable iff if there exist constants α > 0, K > 0 with$\|x(t,s)\| \le Ke^{\alpha(t-s)}$,**0 ≤ s ≤** t<∞.

20. Prove that the time variant system $\dot{x} = Ax + Bu,$ x $\in R^m$, u $\in R^n$ is controllable if and only if there exist the m × n matrices K_1, K_2 for which the matrix $I - e^{-(A+BK_2)T}e^{((A+BK_1)T}$ is invertible.

21. Show that if x(t) and p(t) are the solutions of the canonical equations $\begin{bmatrix} \dot{x}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & -S(t) \\ -Q(t) & -A^*(t) \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix}$ and p(t) = K(t)x(t)for all t $\in$[0, T] and all x(t), then K(t) must satisfies the equation $\dot{K}(t) + K(t)A(t) + A^*(t)K(t) - K(t)S(t)K(t) + Q(t) = 0.$

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