(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2020 ONLY)

20PMS416

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : JULY-2022

M.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: IV **TIME : 3 HOURS**

OPERATOR THEORY

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. K1

1. Let X be a normed linear space & X' be its dual space. Then $\dim X$ ____ $\dim X'$.

   a) $\leq$ b) $=$ c) $\geq$ d) none

2. l^p and $L^p[a,b]$ are ____________

   a) same b) reflexive c) non- reflexive d) none

3. $K(X,Y)$ is a subspace of ________

   a) $B(X,Y)$ b) $\emptyset$ c) X d) none

4. Let X be a Banach space and $A \in B(X)$. If $R(A)$ is _____ in X, then $0 \in \sigma_{app}(A)$

   a) closed b) open c) not closed d) not open

5. If A is a linear operator from $A \to X$, then $< A^*y, x > = < y, Ax >$. Is it true?

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. K2

6. How are the elements of $NBV[a,b]$ called?
7. When is a normed linear space X is reflexive?
8. When the function $A: l^p \to l^p$ defined by $(Ax)(i) = \lambda_i x(i)$ is compact?
9. What is the eigen space of A corresponding to eigenvalue λ?
10. What is $\rho(A)$?

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K3

11. a) A normed linear space is separable if the dual is separable. Prove

(OR)

b) Can the dual of a non-separable normed linear space be homeomorphic to a separable space? Justify your answer.

12. a) Let X and Y be normed linear spaces. If $T: X \to Y$ is a surjective linear isometry then $T^{`}: Y^{`} \to X^{`}$ is a surjective linear isometry. Prove.

(OR)

b) Prove that a Banach Space is reflexive iff its dual is reflexive.

(CONTD…..2)

13.a) If $A \in K(X,Y)$ and $B \in K(Y,Z)$ and if one of them is compact, then $BA \in K(X,Z)$. Prove

(OR)

b) For every bound sequence (x_n) in $L^1[a,b]$, the sequence (kx_n) has a convergent sequence in $(C[a,b], \|.\|_\infty)$.Prove

14.a) Prove that if X is a normed linear space and $A: X \to X$ is a linear operator**,** then $\sigma_{eig}(A) \subseteq \sigma_{app}(A)$. If X is finite dimensional, then **$\sigma_{eig}(A) = \sigma_{app}(A)$**

(OR)

b) Suppose X is a Banach space. $A \in B(X)$ and $\lambda \in K$**.** Then $\lambda \in \sigma(A)$ if and only if either $\lambda \in \sigma_{app}(A)$ or $R(A - \lambda I)$ is not dense in X. **Prove**

15.a) Let X, Y be inner product spaces. $A: X \to Y$ be a linear operator such that the adjoint A^* exists. If $A \in B(X,Y)$, then $A^* \in B(Y,X)$ **and** $\|A^*\| = \|A\|$**,** $\|A^*A\| = \|A\|^2$. Prove

(OR)

b) Let X be a Hilbert space, $A \in B(X)$. Prove the following:

(i). A is normal if and only if $\|Ax\| = \|A^*x\| \; \forall \, x \in X$.

(ii). A is unitary if and only if A is surjective & $\|Ax\| = \|x\| \; \forall \, x \in X$. In particular, if A is unitary then $\|A\| = 1$.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS .

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 & K5)

16. Prove that the dual of $l^p(n)$ is linearly isomorphic to $l^q(n)$**.**

17. State and prove Fundamental theorem on Lebeque integral.

18. State and prove Schur's Lemma.

19. Let X and Y be normed linear spaces. Then Prove
 (i). If $A \in K(X,Y)$, then $A' \in K(Y',X')$.
 (ii). Converse of (i) holds only if Y is a banach space.

20. Let X be a banach space and $A \in B(X)$. Prove the following:
 (i). The map $\lambda \to R(\lambda)$ is continuous on $\rho(A)$.
 (ii). For every $\lambda \in \rho(A), \lim_{\mu \to \lambda} \frac{R(\mu)-R(\lambda)}{\mu-\lambda} = [R(\lambda)]^2$.
 (iii). For every $f \in (B(X))'$, the map $\phi_f : \rho(A) \to \mathbb{C}$ defined by $\phi_f(X) = f(R(\lambda)), \lambda \in \rho(A)$ is differentiable in $\rho(A)$.

21. State and prove Closed range theorem.

****