(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS207

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : MAY - 2023
COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**
SEMESTER: II **TIME : 3 HOURS**

PARTIAL DIFFERENTIAL EQUATIONS

SECTION-A (10 x 1=10 Marks)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **K1**

1. The equations f(x,y,z,p,q) = 0 and g(x,y,z,p,q)= o are compatible on a domain if______________
   a) $J = \frac{\partial(f,g)}{\partial(p,q)} \neq 0 \; on\; D$ and p= $\phi(x,y,z)$ $q = \psi(x,y,z)$
   b) $J = \frac{\partial(f,g}{\partial(p,q)}$ = 0 on D and $p = \phi(x,y,z)$ q = $\psi(x,y,z)$
   c) $J = \frac{\partial\,(p,q)}{\partial(x,y)} = 0$ on D and $p = \phi\,(x,y,z)$ $q = \psi\,(x,y,z)$
   d) $J = \frac{\partial(p,q)}{\partial(f,g)} \neq 0$ on D and
2. F (D,D') e^{ax+by} = ______________.
   a) F (b,a) e^{ax+by} b) F(a,b) e^{ax+by} c) F(a,b) d) F(b,a)
3. The Hankel transform is given by___________________.
   a) $\int_0^\infty rV(r,z)J_{0\,(\xi r)dr}$ b) $\int_0^\infty rV(z,f)J_{0\,(\xi r)dr}$ c) $\int_0^\infty rV(r,z)J_{0(x)\,dr}$ d) $\int_0^\infty rV(r,z)J_{0\,(z,r)dr}$ b)
4. If f is a continuous on the boundary S of a finite region V, the problem of finding $\psi(x,y,z)$ such that $\nabla^2\psi = 0$ within V and its normal derivative $\frac{\partial\psi}{\partial n} = f$ $on\ S$ is called_____________.
   a) Newmann problem b) Dirichelts problem c) Cauchy's problem d) Churchill problem
5. The equation $\partial^2 z/\partial x^2 + \frac{\partial^2 z}{\partial y^2} = 0$ is called_____________.
   a) Laplace equation b) Cauchy equation c) Wave equation d) Diffusion equation

ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES. **K2**

6. Write the subsidiary equations for Charpit's method.
7. Define complementary function and particular integral.
8. Write the Hankel transform.
9. Define a family of equipotential surface.
10. Write the Vibrating membrane equation.

SECTION-B (5X 3=15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **K3**

11. a) Show that the equations $xp = yq$ and $z(xp + yq)$ are compatible.

(OR)

b) Find the complete integral of p + q = pq

12 .a) Find the particular integral of $(D^2 - D')z = 2y - x^2$

(OR)

b) Show that, if the operator F (D, D') is reducible, the order in which the linear factors occur is unimportant.

13. a) Write the procedures to be followed in applying the theory of Integral transforms to the solution of PDE.

(OR)

b) Find the solution of the one dimensional wave equation by the method of separation of variables.

14. a) Derive the elementary solution of Laplace equation.

(OR)

b) Show that in the two-dimensional case, Neumann problem can be reduced to the Dirichlet problem.

15.a) Derive the elementary solution of Diffusion equation.

(OR)

b) Find the approximate values of the first two Eigen values of a square membrane of side two.

SECTION-C (5 x 5= 25 Marks)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K4 & K5

16.a) Solve the nonlinear PDE $z^2(1+p^2+q^2)=1$

(OR)

b) Solve the PDE $p^2x+q^2y=z$ by Jacobi method.

17 .a) Solve the equation $\frac{\partial^4 z}{\partial 4} - \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^2 z}{\partial x^2 \partial y^2}$.

(OR)

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ to a canonical form.

18. a) Solve $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

(OR)

b) Derive the solution of the equation $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$

19. a) A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity of the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

(OR)

b) Show that the surface $x^2 + y^2 + 2cx^{\frac{2}{3}}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

20. a) Obtain D' Alembert's solution for the one-dimension wave equation.

(OR)

b) The faces $x = 0, x = a$ of an infinite slab are maintained at zero temperature. The initial temperature in the slab is described by the equation $\theta = f(x), (0 \le x \le a)$. Determine the temperature at a subsequent time t.

ETHICAL PAPER

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