(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS4E1

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY - 2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: IV **TIME : 3 HOURS**

MATHEMATICAL METHODS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER ALL THE QUESTIONS.
MULTIPLE CHOICE QUESTIONS. K1

1. Resolvent kernel is $\Gamma(s,t:\lambda) =$ ________.
   a. $\sum_{m=1}^{\infty} k_m(s,t)$ b. $\sum_{m=1}^{\infty} \lambda^{m-1} k_m(s,t)$ c. $\sum_{m=1}^{\infty} \lambda k_m(s,t)$ d. $\sum_{m=1}^{\infty} \lambda^{m} k_m(s,t)$

2. Abel integral equation is $(s) =$ _____ , $0 < \alpha < 1$.
   a. $\int_0^s \frac{g(t)}{(s-t)^\alpha}\, dt$ b. $\int_0^s \frac{g(t)}{(s-t)}\, dt$ c. $\int_0^s (s-t)^\alpha\, g(t)\, dt$ d. $\int_0^s g(t)\, dt$

3. Euler's equation is _______ .
   a. $Fy + \frac{d}{dx} Fy = 0$ b. $Fy + \frac{d}{dx} Fy' = 0$ c. $Fy - \frac{d}{dx} Fy' = 0$ d. $Fy - \frac{d}{dx} Fy = 0$

4. For a strong minimum, the condition $E \geq 0$ is replaced by __________.
   a. $F_{y'y'}(x,y,q) \geq 0$ b. $F_{y'y'}(x,y,q) \leq 0$ c. $F_{y'}(x,y,q) \geq 0$ d. $F_{y'}(x,y,q) \leq 0$

5. ______ method is frequently employed for exact or approximate solutions of problems in mathematical physics.
   a. Laplace b. Ritz c. Euler d. Leibnitz

ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES. K2

6. State Fredholm Alternative theorem.
7. Define initial value problem.
8. Write the fundamental lemma of the calculations of variations.
9. Write the Hamilton – Jacobi equation.
10. Write about Galerkin's method.

SECTION – B (5 X 4 = 20MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K3

11. a. Find the resolvent kernel for the integral equation,

$$g(s) = f(s) + \lambda \int_{-1}^{1} (st + s^2t^2)\, g(t)\, dt$$

(OR)

b. Solve the integral equation **:** $g(s) = f(s) + \lambda \int_0^1 e^{s-t}\, g(t)\, dt$

(CONTD……2)

12.a. Reduce the initial value problem $y''(s) + \lambda\, y(s) = F(s), 0 \le s, y(0) = 1, y'(0) = 0$ to a Volterra integral equation.

(OR)

b. Solve the integral equation **:** $s = \int_0^s \frac{g(t)\, dt}{(s-t)^{1/2}}$.

13.a. Find the extremal of the functional $v[y(x)] = \int_0^1 \left(1 + y''^2\right) dx$ where

$$y(0) = 0, y'(0) = 1, y(1) = 1, y'(1) = 1$$

(OR)

b. On what curves can the functional $v[y(x)] = \int_0^1 [(y')^2 + 12xy]\, dx$ with $y(0) = 0,\ y(1) = 1$ be extremized?

14. a) Test for the extremum the functional $\int_0^a (6y' - y'^4 + yy')\, dx$, y (0) = 0 ; y (a) = b ; a > 0 and b > 0 in the class of continuous functions with continuous first derivative.

(OR)

b) Test for an extremum the functional $v[y(x)] = \int_0^a (y'^2 - y^2)\, dx$, a>0 ; $y(0) = 0, y(a) = 0$.

15.a. Explain Euler's finite difference method.

(OR)

b. Find a solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x, y)$ inside the rectangle $\mathcal{D} : 0 \le x \le a, 0 \le y \le b$, that vanishes on the boundary of D.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS.
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS.
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Solve the integral equation $g(s) = f(s) + \lambda \int_0^s e^{s-t}\, g(t)\, dt$, and evaluate the resolvent kernel.

17. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s + t)\, g(t)\, dt$ and find the eigen values.

18. Solve the integral equation

$$f(s) = \int_a^s \frac{g(t)\, dt}{(\cos t - \cos s)^{1/2}}, 0 \le a < s < b \le \pi.$$

19. Find the extremals of the functional

$$v[y(x), z(x)] = \int_0^{\pi/2} [y'^2 + z'^2 + 2yz]\, dx, \text{ y}(0) = 0, y(\pi/2) = 1, z(0) = 0,\ z(\pi/2) = -1.$$

20. Test for an extremum the functional $v = \int_0^a y'^3(x)\, dx$, where

$$y(0) = 0, y(a) = b, a > 0, b > 0$$

21. Investigate for an extremum the functional . $v[z(x, y)] = \int_{-a}^{a}\int_{-b}^{b} \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 - 2z\right] dx\, dy$; on the boundary of the integration domain z=0. The integration domain is a rectangle $-a \le x \le a$; $-b \le y \le b$.

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