(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22UPS2A2

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : MAY-2023

COURSE NAME: B.Sc.-PHYSICS **MAXIMUM MARKS: 50**
SEMESTER: II **TIME : 3 HOURS**

PART - III

ANCILLARY MATHEMATICS FOR PHYSICS - II

SECTION – A **(10 X 1 =10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS: [K1]

1. $L\{1\} =$ -------
   a) $\frac{1}{s}$ b) $-\frac{1}{s}$ c) s d) s^2
2. $L^{-1}\left(\frac{1}{s^2+a^2}\right) =$-------
   a) sin at b) $\frac{\sin at}{a}$ c) $\frac{\cos at}{a}$ d) cos at
3. If $\vec{F}$ is solenoidal then $\nabla.\vec{F} =$------
   a) zero b) $\nabla^2 c$ c) constant d) one
4. $\nabla X\,\nabla\varphi$=-------
   a) one b) infinity c) constant d) zero
5. A scalar function φ satisfying the condition $\nabla^2\varphi = 0$ is called the ------ function.
   a) harmonic b) differential c) integral d) Green's

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **[K2]**

6. Define hyperbolic function.
7. Write the Laplace inverse formula for $L^{-1}\left(\frac{s}{s^2-a^2}\right)$.
8. Define irrotational.
9. Define Line integral.
10. State Gauss divergence theorem.

SECTION –B **(5 X 3 = 15 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **[K3]**

11. a) If sin (A + i B) = x + iy prove that i) $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$, ii) $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$.

(OR)

b) Find the Laplace transform $\sin^3 2t$.

12. a) Find the inverse transform of log $\left(\frac{1+s}{s}\right)$.

(OR)

b) Find the inverse transform of log $\left(\frac{1+s}{s}\right)$.

13. a) If $\varphi(x,y,z)=x^2y+y^2x+z^2$ find $\nabla\varphi$ at the point (1, 1, 1).

(OR)

b) Show that the vector $\vec{F} = 3y^4z^2\,\vec{i} + 4x^3z^2\,\vec{j} - 3x^2y^2\,\vec{k}$ is solenoidal.

14. a) If $\vec{F} = 3xy\,\vec{i} - y^2\,\vec{j}$, find $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2) .

(OR)

b) Evaluate $\iiint_V \vec{F}\,dv$ where $\vec{F} = 2xz\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$ and V is the volume of the region enclosed by the cylinder $x^2 + y^2 = a$ between the planes z = 0, z = c.

15. a) Using divergence theorem, evaluate $\int_S \vec{F}.\vec{n}\;dS$ where $\vec{F} = 4xz\,\vec{i} - y^2\,\vec{j} + yz\,\vec{k}$ and S is the Surface of the cube bounded by the planes x =0, x = 2, y = 0, y = 2, z = 0 and z =2.

(OR)

b) Find the area of the ellipse $x = a\cos\theta$, $y = b\sin\theta$.

SECTION –C (5X5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K4 & K5

16. a) Expand $\cosh^8\theta$ in terms of hyperbolic cosines of θ.

(OR) **[K4]**

b) Find the Laplace transform of $\sin^2 t\cos^3 t$.

17. a) Using Laplace transform solve $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=e^{-t}$ given y = 0, $\frac{dy}{dt}=1$ when t = 0.

(OR) **[K5]**

b) Find the Laplace inverse of $\frac{2(s+1)}{(s^2+2s+2)^2}$.

18. a) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is irrotational and solenoidal.

(OR) **[K4]**

b) If $\vec{F} = xz^3\,\hat{i} - 2xyz\,\hat{j} + xz\,\hat{k}$ find div $\vec{F}$ and curl $\vec{F}$ at (1, 2, 0).

19. a) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force given by $\vec{F} = (2x-y+z)\,\vec{i} + (x+y-z^2)\,\vec{j} + (3x-2y+4z)\,\vec{k}$ is the field conservative?

(OR) [K5]

b) Evaluate $\iint_S \vec{F}.\vec{n}\,dS$ where $\vec{F} = 18z\,\vec{i} - 12\,\vec{j} + 3y\,\vec{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first quadrant.

20. a) Verify Green's theorem in the plane $\oint_C (xy+y^2)dx + x^2dy$ where C is the chord curve of the region bounded by y = x and $y = x^2$.

(OR) [K4]

b) Compute $\int_C (xy-x^2)dx + x^2ydy$ over the triangle bounded by the lines y = 0, x = 1, y = x and verify Green's theorem.

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