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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2020 ONLY)

20UMS614 / 20UMA614

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS :MAY - 2023

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: VI **TIME : 3 HOURS**

PART - III

REAL ANALYSIS II

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If f is differentiable at a point c then ___________________.
   a. f converges to c b. f has a point of discontinuity at c
   c. f is continuous at c d. $f'(c) = 0$

2. If f is monotonic on [a,b] then the set of discontinuities of f is _______________
   a. bounded b. finite c. infinite d. countable

3. The Riemann Stieltjes integral sum of f with respect to α is _________________
   a. S (P,f,α) = $\sum_{k=1}^{n} f(t_k)\Delta a_k$ b. S (P,f,α) = $\sum_{k=1}^{n} f(t_k)a_k$
   c. S (P,f,α) = $\sum_{k=1}^{\infty} f(t_k)\Delta a_k$ d. S (P,f,α) = $\sum_{k=1}^{n} f(t_k)\Delta f_k$

4. If f and $|f|$ ∈R(α) and α is increasing on [a,b] then____________.
   a. $\left|\int_a^b f(x)d(x)\right| \leq \int_a^b |f(x)|\, d\alpha(x)$ b. $\left|\int_a^b f(x)d\alpha(x)\right| \geq \int_a^b |f(x)|\, d\alpha(x)$
   c. $\left|\int_a^b f(x)d\alpha(x)\right| \geq \int_a^b |f(x)|\, d(x)$ d. $\left|\int_a^b f(x)d\alpha(x)\right| \leq \int_a^b |f(x)|\, d\alpha(x)$

5. The value of $\int_e^f h(x)\, d\alpha(x) = c\int_e^f d\alpha(x)$= c[α(f) – α(e)]becomes zero if ____________
   a. α(f) – α(e) = 0 b. e = b c. both a and c d. $|\alpha(f) - \alpha(e)| > \in$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. When is a function said to be differentiable at a point ‘a’?
7. What is the additive property of total variation?
8. Give an example for a step function.
9. Write the upper Stieltjes sum of f with respect to α for the partition P.
10. State First mean- value theorem on Riemann Stieltjes Integrals.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If f and g are functions defined on (a,b) are differentiable at c, then show how $f.g$ will also be differentiable at c.

(OR)

b) Show that the composite function gof is differentiable if f and g are differentiable at c and f(c) respectively.

(CONTD…..2)

12. a) If *f* is a bounded variation on [a,b] and V is a function on [a,b] defined as $V(x) = V_f(ax)$, if $a < x \leq b$, $V(a) = 0$, then show that V and V-*f* are increasing functions on [a,b]

(OR)

b) Show that if a function *f* is bounded variation defined on [a,b] if and only if *f* can be expressed as a difference of two increasing functions

13. a) Derive Euler's Summation formula.

(OR)

b) Prove the theorem of integration by parts

14. a) How will you prove U (P', *f*, α) ≤ U(P, *f*,α) and L (P', *f*, α) ≥L (P, *f*,α) if P' is finer than P if αis increasing on [a,b]

(OR)

b) If αis increasing on [a,b], f and g ∈R(α)and f(x) ≤g(x) for all x in [a,b] then show that $\int_a^b f(x) \leq \int_a^b g(x)$

15. a) Prove the theorem on Second mean- value theorem on Riemann Stieltjes Integrals

(OR)

b) If g∈ R on [a,b] and α is a continuous function on [a,b] with derivative α' as Riemann integrable on [a,b] then show the existence of the integral $\int_a^b g(x)\, d\alpha(x) = \int_a^b g(x)\, \alpha'(x) dx$

SECTION - C (4X10= 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS.
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. State and prove the Mean value theorem for derivatives

17. Prove the intermediate value theorem for derivatives with suitable arguments.

18. Let f be a bounded variation on [a,b], if $x \in$ (a,b], $V(x) = V_f(ax)$ and V(a) = 0. Is every point of continuity of f is a point of continuity of V? If so substantiate the same.

19. If f∈ R (α) on [a,b] and α has continuous derivatives α'on [a,b], then justify the existence of $\int_a^b f(x)\, \alpha'(x) dx$ and $\int_a^b f(x)\, d\alpha(x) = \int_a^b f(x)\, \alpha'(x) dx$

20. If αis increasing on [a,b] then justify the equivalence of the given three statements
    (i) f ∈ R(α) on [a,b]
    (ii) f satisfies Riemann's condition with respect to α on [a,b]
    (iii) $\underline{I}$ (, *f*, α) = I (*f*, α)

21. Substantiate the theorem on second fundamental theorem of integral calculus

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